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TECHNICAL REPORT K-80-3

DOCUMENTATION FOR LMVDPILE PROGRAM

by

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June 1980

Final Report

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Prepared for U. S. Army Engineer Division, Lower Mississippi Valley
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The primary work reported here consists of consolidating the rigid cap pile analysis programs of the U. S. Army Engineer Districts, St. Louis and New Orleans. The new program, LMVDPILE, is documented with example problems in this report. The work was performed at the request of the Lower Mississippi Valley Division and provides the capability of analyzing two-dimensional or three-dimensional pile foundations according to Division guidelines. The report includes discussions of factors influencing pile group behavior and of the

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20. ABSTRACT (Continued).

analytical procedure, a user's guide, and several example problems for the pile analysis program LMVDPILE. Also included are two appendices. Appendix A describes the computer program PILESTF which computes the pile head stiffness coefficients for piles in soils with varying moduli. Appendix B describes the computer program FDRAW which is an interactive graphics post-processor. Each appendix includes a general introduction, a user's guide, and example problems.

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PREFACE

The study reported herein was performed at the U. S. Army Engineer Waterways Experiment Station (WES) as part of the support provided by the Computer-Aided Design Group (CADG) of the Automatic Data Processing (ADP) Center to the U. S. Army Engineer Division, Lower Mississippi Valley (LMVD).

The work involved consolidation of two existing pile analysis programs, one from the St. Louis District and the other from the New Orleans District. This work was performed by Ms. Deborah K. Martin, formerly of CADG. The computer program PILESTF which is described in Appendix A was coded and documented by Dr. William P. Dawkins, Consultant, Oklahoma State University, Stillwater, Okla. PILESTF computes the pile head stiffness coefficients for piles in soils with varying moduli. The computer program FDRAW which is described in Appendix B was coded and documented by Mr. John Jobst of the St. Louis District. FDRAW is an interactive graphics post-processor program that can display pile geometry, resultant axial forces, pile loading factors, and elastic center diagrams. The authors thank Dr. Dawkins and Mr. Jobst for their contributions to this work.

This report was written by Ms. Martin, Mr. H. Wayne Jones, CADG, and Dr. N. Radhakrishnan, Special Technical Assistant, ADP Center. Technical contact at the St. Louis District was Mr. Thomas J. Mudd and at the New Orleans District was Mr. C. W. Ruckstuhl. The authors thank Mr. Mudd, Mr. Ruckstuhl, and several of their co-workers for their technical guidance.

The study was monitored at LMVD by Mr. Victor Agostinelli, Technical Engineering Branch. The work was done under the general supervision of Mr. D. L. Neumann, Chief of the ADP Center.

COL J. L. Cannon, CE, and COL N. P. Conover, CE, were Directors and Mr. F. R. Brown was Technical Director of WES during the performance of the work and the preparation of the report.

CONTENTS

CONVERSION FACTORS, U. S. CUSTOMARY TO METRIC (SI) UNITS OF MEASUREMENT

U. S. customary units of measurement used in this report can be converted to metric (SI) units as follows:

Multiply	By	To Obtain
feet	0.3048	metres
inches	2.54	centimetres
kips (1000 lb force)	4.448222	kilonewtons
kips (force) per square foot	47.88026	kilopascals
pounds (mass)	0.45359237	kilograms
pounds (force) per square inch	6.894757	kilopascals
pounds (mass) per cubic foot	16.01846	kilograms per cubic metre
pounds (mass) per cubic inch	0.02768	kilograms per cubic centimetre
square inches	6.4516	square centimetres

DOCUMENTATION FOR LMVDPILE PROGRAM

PART I: INTRODUCTION

Background

1. Many Corps of Engineers offices use the Hrennikoff (1950) method to analyze pile foundations. This method was originally proposed for analyzing two-dimensional pile foundations but has been refined and extended by Saul (1968) for three-dimensional foundations.

2. The U. S. Army Engineer Districts, St. Louis and New Orleans, each use a different version of a pile analysis computer program, but both use the Hrennikoff method. The Technical Engineering Branch of the Lower Mississippi Valley Division (LMVD) was interested in standardizing the two Districts' programs into one program, LMVDPILE, that would include all options from both programs. The work described herein was performed at the request of LMVD. The result of this work provides the capability of analyzing two-dimensional or three-dimensional pile foundations according to the LMVD guidelines.

Scope

3. Factors influencing pile group behavior, the analytical procedure, a user's guide, and several example problems for the pile analysis program LMVDPILE are presented herein. User's guides for a pre-processor routine called PILESTF that can compute the pile head stiffness matrix for a pile in a layered soil mass and for an interactive graphics post-processor program, FDRAW, are also included.

PART II: FACTORS INFLUENCING PILE GROUP BEHAVIOR*

4. Foundation piles are supporting structural members which transfer loads from the structure to the subsoil. Adequate design will insure that excessive deflections and stresses in the "structure-pile-soil system" will not occur. Generally, it is not a difficult task to determine the loads acting on the pile foundation from the structure. However, the distribution of the loads from the piles to the soil is a highly indeterminate and sometimes nonlinear problem. This leads to complex solutions of the pile-soil interaction problem. Many conditions affect the resistance of the pile foundation to movement and the transfer of loads from the structure to the pile-soil medium (Mudd 1969).

Factors that Influence Capacity of Pile Foundations

5. The capacity of a pile foundation can be defined as its ability to resist applied loads without exceeding certain allowable deflections or stresses. The following variables should be considered during analysis of the load-carrying capacity of the soil-pile medium.

Subgrade modulus

6. A subgrade modulus can be employed to relate the lateral, axial, and rotational resistance of the pile-soil medium to displacements. The subgrade modulus is a function of the nature of the loading, the elasticity of the pile, and the stress-strain characteristics of the surrounding soil. Therefore, the determination of the subgrade modulus depends on the nonlinear and nonelastic, pile-soil stress-strain relationship characteristics. The load-carrying capacity of the foundation is dependent on these nonlinear and nonelastic characteristics.

Fixity

7. The fixity of the pile head into the pile cap influences the load-carrying capacity of a pile foundation. Generally, fixing the pile heads completely rather than pinning them into the pile cap will double

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* Major portions of Part II are extracted from Mudd (1969).

the lateral stiffness of the foundation. Thus the fixed pile can carry twice the lateral load with the equivalent deflection as the pinned pile foundation.

Batter

8. The direction and slope of batter affect the subgrade modulus. Murthy (1964) has shown with model pile tests that piles battered upstream are more resistant to lateral loads than piles battered downstream. A pile battered upstream is defined as having its tip further upstream than its top, and a pile battered downstream as having its tip further downstream than its top.

Group effect

9. Close spacing of piles will affect the lateral and vertical resistance of adjacent piles within a pile group. Prakash (1962) has shown that piles spaced from three to eight pile diameters apart (normal to the load) cause a reduction in the lateral capacity of the group. A pile spacing of less than three diameters decreases the stiffness of the pile group by about one half of the sum of the same number of isolated piles. The group effect can be accounted for by reducing the subgrade modulus by an appropriate factor. Similar effects have been noted for the axial capacity of group friction piles.

Position in group

10. Prakash has also shown that the position of the pile in a group affects its individual stiffness influence coefficients. He has shown that a pile in the interior of the group would be more flexible than one on the perimeter. This is due to the interference of the zone of influence of the pile by adjacent piles when these zones overlap.

Stiffness of pile cap

11. The stiffness of the pile cap will influence the distribution of the structural loads to the individual piles. A multicolumn bent can be approximated as having a rigid top if the cap is 10 or more times stiffer than the columns. Therefore a rigid pile cap can generally be assumed for gravity-type hydraulic structures. If the cap is less than rigid, then the problem becomes one of achieving compatibility between pile-head displacements and the structure deformation. The program

SAPIV has been modified to include a pile element (Jones and Radhakrishnan 1975). This will allow the analysis of flexible pile caps if necessary.

Nature of loading

12. The different conditions of static, cyclic, dynamic, and transient loadings affect the ability of the pile foundation to resist applied forces.

13. Cyclic loading (repeated application of a static load) causes a greater deflection than the application of a sustained static load of the same magnitude. In some pile tests the application of cyclic loading doubles the deflection over that of the application of a single static load for a given level of loading (U. S. Army Engineer District, Little Rock 1964).

14. Piles subjected to vibratory loads may produce greater pile displacements than piles subjected to static loads. At present, little is known of the quantitative effect vibrations may have on the load-carrying capacity of pile-founded structures.

15. If tension and compression piles are present in a foundation, the tension pile may have a reduced load-carrying capacity from that of the compression pile for equivalent deflections. Also, the tension pile may have less lateral stiffness than an equivalent compression pile.

Pile driving

16. Driving piles in a group increases the density of the soil within and around a pile group. Consequently the stiffness of the soil may increase by driving piles in closely spaced groups. Although tests on a single pile within a group may indicate an increased stiffness due to pile driving, the pile group as a whole may not reflect this increased stiffness. A larger zone of stressed soil may not be favorably affected by pile driving. Thus deflections larger than anticipated may result. Therefore, lateral load tests on a single pile in a large group of piles may indicate liberal stiffness coefficients.

Water table and seepage pressures

17. The position of the water table affects the lateral subgrade modulus. Effects of submergence have been accounted for by some

designers by reducing the lateral subgrade modulus by the ratio of submerged unit weight of the soil to its dry unit weight. An additional load on the pile foundation can be caused by seepage pressures under structures that support unbalanced water loads. These seepage pressures also may affect the subgrade modulus of the soil.

Sheet pile cutoffs

18. Sheet pile cutoffs inclosing the pile group may change the distribution of stress in the soil, affecting the load-carrying capacity of the foundation.

Length of pile

19. The length of a pile will affect the lateral and axial subgrade modulus. The lateral subgrade modulus is different for short rigid piles that act as poles and long flexible piles that act in flexural bending. Piles can be considered to act in the flexural mode if the nondimensional length L/T is greater than 5, as defined by Reese and Matlock (1960).

Conclusion

20. All these factors must be considered if a valid analysis of pile foundations is to be accomplished. The effects of most of these variables can be accounted for in the analysis by appropriate changes in the value of the subgrade modulus obtained from pile test data of a single free pile.

PART III: PROCEDURE FOR THE ANALYSIS OF PILE FOUNDATIONS*

21. A general direct stiffness analysis method for three-dimensional pile foundations has been presented by Saul (1968), which expands the Hrennikoff (1950) method from two dimensions to three. This method appears to be general, provided the designer has an understanding of matrix methods and structure-soil-pile interactions and an electronic computer available to perform the computations. The method uses exact numerical analysis solutions for solving the assumed soil-pile model. However, the designer must have an adequate representation of soil-pile interaction for input to the method. Various factors that influence the soil-pile interaction have been discussed in Part II.

The General Model

22. A generalized model of the structure-pile system can be described as a rigid body supported by sets of springs which represent the actions of the pile forces on the structure when the structure undergoes unit displacements. It is assumed that the pile head loading for any single pile in a batter group may be resolved into a combination of axial load, bending moment, shear, and torque. Also, each of these components can be represented by a proper spring constant and results added vectorially to obtain the total movement of the pile head. This method of analysis only considers the effect the piles have on the pile cap at the top of the pile; i.e., each pile can be replaced by the proper elastic spring restraints at the pile cap. The assumptions required by this method are:

- a. A rigid piling cap.
- b. Elastic behavior of the system.
- c. Effects of displacement for six degrees of freedom in a three-dimensional analysis or for three degrees of freedom in a two-dimensional analysis can be superimposed.

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* Major portions of Part III are extracted from Mudd (1969).

23. This method can also account for:

a. Any degree of fixity of any pile with the pile cap.

b. Piles with different bending stiffness about their principal axes.

c. Any degree of linear (elastic) torsional, axial, or lateral resistance of any pile in the foundation.

d. Any position and batter of piles in the foundation.

e. Piles of different sizes or materials in the foundation.

24. If the restrictions as stated in paragraph 22 are not allowed, then the response of the system is nonlinear, and a closed form solution cannot be achieved. However, it is possible to include these in some type of iterative procedure.

Analysis

Elastic pile constants, three-dimensional system

25. Each pile has six degrees of freedom in a three-dimensional system: two lateral, one axial, two moment, and one torsional. The forces and displacements along the pile axes are shown in Figure 1 in which axes U_1 and U_2 are principal axes of inertia and axis U_3 coincides with the longitudinal axis of the piling. In a two-dimensional system, each pile has three degrees of freedom: one lateral, one axial, and one moment. Figure 2 shows the forces and displacements along the pile axes. The pile forces can be equated to the pile displacements by the expression

$$\{F\}_i = \{b\}_i \{X\}_i \tag{1}$$

such that b_i are the individual pile stiffness influence coefficients called the elastic pile constants. The $\{b\}_i$ matrix for a three-dimensional system can be defined for the i^{th} pile as

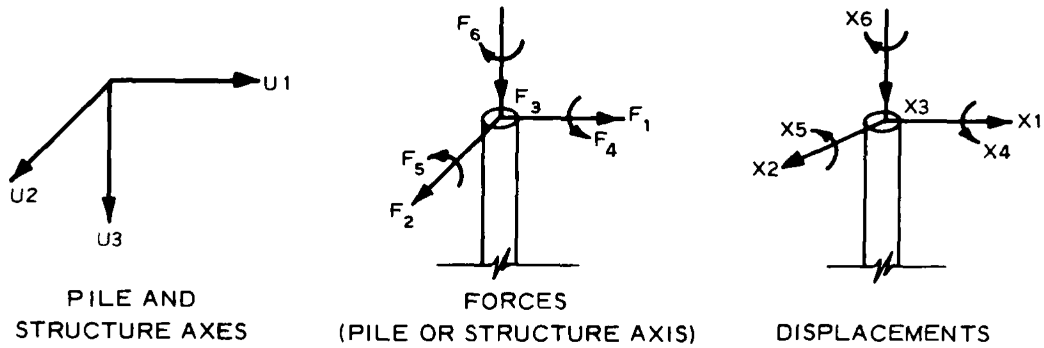

Figure 1. Coordinate system for three-dimensional system

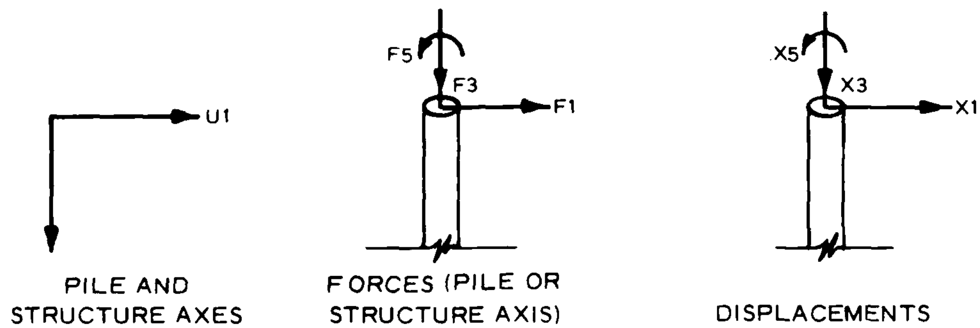

Figure 2. Coordinate system for two-dimensional system

$$\{b\}_i = \begin{Bmatrix} b_{11} & 0 & 0 & 0 & b_{15} & 0 \\ 0 & b_{22} & 0 & b_{24} & 0 & 0 \\ 0 & 0 & b_{33} & 0 & 0 & 0 \\ 0 & b_{42} & 0 & b_{44} & 0 & 0 \\ b_{51} & 0 & 0 & 0 & b_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & b_{66} \end{Bmatrix} \qquad (2)$$

26. The elastic pile constants are defined as follows:

b_{11} is the force required to displace the pile head a unit distance along the U_1-axis, FORCE/LENGTH

b_{22} is the force required to displace the pile head a unit distance along the U_2-axis, FORCE/LENGTH

b_{33} is the force required to displace the pile head a unit distance along the U_3-axis, FORCE/LENGTH

b_{44} is the moment required to displace the pile head a unit rotation around the U_1-axis, FORCE-LENGTH/RADIAN

b_{55} is the moment required to displace the pile head a unit rotation around the U_2-axis, FORCE-LENGTH/RADIAN

b_{66} is the torque required to displace the pile head a unit rotation around the U_3-axis, FORCE/RADIAN

b_{15} is the force along the U_1-axis caused by a unit rotation of the pile head around the U_2-axis, FORCE/RADIAN

$-b_{24}$ is the force along the U_2-axis caused by a unit rotation of the pile head around the U_1-axis, FORCE/RADIAN (Note: The sign is negative.)

b_{51} is the moment around the U_2-axis caused by a unit of displacement of the pile head along the U_1-axis, FORCE-LENGTH/LENGTH

$-b_{42}$ is the moment around the U_1-axis caused by a unit displacement of the pile head along the U_2-axis, FORCE-LENGTH/LENGTH (Note: The sign is negative.)

Elastic pile constants,
two-dimensional system

27. The $\{b\}_i$ matrix for a two-dimensional system can be defined for the i^{th} pile as

$$\{b\}_i = \begin{Bmatrix} b_{11} & 0 & b_{13} \\ 0 & b_{22} & 0 \\ b_{31} & 0 & b_{33} \end{Bmatrix} \qquad (3)$$

28. The elastic pile constants are defined as follows:

b_{11} is the force required to displace the pile head a unit distance along the U_1-axis, FORCE/LENGTH

b_{22} is the force required to displace the pile head a unit distance along the U_3-axis, FORCE/LENGTH

b_{33} is the moment required to displace the pile head a unit rotation around the U_2-axis, FORCE-LENGTH/RADIAN

b_{13} is the force along the U_1-axis caused by a unit rotation of the pile head around the U_2-axis, FORCE/RADIAN

b_{31} is the moment around the U_2-axis caused by a unit displacement of the pile head along the U_1-axis, FORCE-LENGTH/LENGTH

29. The elements for the $\{b\}_i$ matrix are symmetric. That is:

$$b_{15} = b_{51}$$

$$b_{24} = b_{42}$$

for a three-dimensional system. For a two-dimensional system,

$$b_{13} = b_{31}$$

<u>Constant soil modulus</u>

30. If it is assumed that the lateral subgrade modulus is constant with depth, then the pile constants for a three-dimensional system can be derived as follows. If

$$\beta_1 = \sqrt[4]{\frac{E_s}{4EI_2}} \ , \ \beta_2 = \sqrt[4]{\frac{E_s}{4EI_1}} \qquad (4)$$

then

$$b_{11} = (1 + DF)\left(\frac{E_s}{2\beta_1}\right) \qquad (5)$$

$$b_{22} = (1 + DF)\left(\frac{E_s}{2\beta_2}\right) \qquad (6)$$

$$b_{33} = K_2\left(\frac{AE}{L}\right) \qquad (7)$$

$$b_{44} = DF\left(\frac{E_s}{2\beta_2^3}\right) \qquad (8)$$

$$b_{55} = DF\left(\frac{E_s}{2\beta_1^3}\right) \tag{9}$$

$$b_{66} = K_4 \tag{10}$$

$$b_{15} = DF\left(\frac{E_s}{2\beta_1^2}\right) \tag{11}$$

$$b_{24} = DF\left(\frac{E_s}{2\beta_2^2}\right) \tag{12}$$

$$b_{51} = b_{15} \tag{13}$$

$$b_{42} = b_{24} \tag{14}$$

where

E_s = lateral subgrade modulus, $FORCE/LENGTH^2$*

E = modulus of elasticity, $FORCE/LENGTH^2$

I_1, I_2 = moment of inertia, $LENGTH^4$, about the U_1 and U_2 axes, respectively

DF = degree of fixity (fraction)

Constants K_2 and K_4 = degrees of pile rigidity under axial and torsional behavior, respectively. K_2 is normally assumed to be 1.0 for bearing piles and 2.0 for friction piles. K_4 is normally assumed to be zero as the torsional behavior of the pile is not well known.

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* The lateral subgrade modulus E_s required in the program input should include width effect of the pile, group effect, cyclic load effect, etc. There are no provisions in the program to internally calculate these effects.

A = cross-sectional area of pile, $LENGTH^2$

L = length of pile, LENGTH

31. For a two-dimensional system, if

$$\beta_1 = \sqrt[4]{\frac{E_s}{4EI_2}} \tag{15}$$

then

$$b_{11} = (1 + DF)\left(\frac{E_s}{2\beta_1}\right) \tag{16}$$

$$b_{22} = K_2\left(\frac{AE}{L}\right) \tag{17}$$

$$b_{33} = DF\left(\frac{E_s}{2\beta_1^3}\right) \tag{18}$$

$$b_{13} = DF\left(\frac{E_s}{2\beta_1^2}\right) \tag{19}$$

$$b_{31} = b_{13} \tag{20}$$

Linearly varying subgrade moduli

32. If it is assumed that the lateral subgrade modulus varies linearly with depth, $E_s = K_s(X_3)$, then the pile constants for a three-dimensional system can be derived as follows. If

$$T_1 = \sqrt[5]{\frac{EI_2}{K_s}}, \quad T_2 = \sqrt[5]{\frac{EI_1}{K_s}} \tag{21}$$

then

$$b_{11} = K_1\left(\frac{EI_2}{T_1^3}\right) \tag{22}$$

$$b_{22} = K_1\left(\frac{EI_1}{T_2^3}\right) \tag{23}$$

$$b_{33} = K_2\left(\frac{AE}{L}\right) \tag{24}$$

$$b_{44} = K_3\left(\frac{EI_1}{T_2}\right) \tag{25}$$

$$b_{55} = K_3\left(\frac{EI_2}{T_1}\right) \tag{26}$$

$$b_{66} = K_4\left(\frac{JG}{L}\right) \tag{27}$$

$$b_{15} = K_5\left(\frac{EI_2}{T_1^2}\right) \tag{28}$$

$$b_{24} = K_5 \left(\frac{EI_1}{T_2^2} \right) \tag{29}$$

$$b_{51} = K_6 \left(\frac{EI_2}{T_1^2} \right) \tag{30}$$

$$b_{42} = -K_6 \left(\frac{EI_1}{T_2^2} \right) \tag{31}$$

where

E = modulus of elasticity, FORCE/LENGTH^2

I_1, I_2 = moments of inertia, LENGTH^4, about U_1 and U_2 axes, respectively

K_s = coefficient of subgrade modulus, FORCE/LENGTH^3

A = cross-sectional area of pile, LENGTH^2

L = length of pile, LENGTH

J = polar moment of inertia, LENGTH^4

G = torsion modulus, FORCE/LENGTH^2

K_1 = lateral fixity coefficient

K_2 = pile axial resistance coefficient

K_3 = rotational fixity coefficient

K_4 = coefficient for torsion

K_5 = fixity coefficient

K_6 = fixity coefficient

33. For a two-dimensional system, if

$$T = \sqrt[5]{\frac{EI}{K_s}} \tag{32}$$

then

$$b_{11} = K_1 \left(\frac{EI}{T^3}\right) \tag{33}$$

$$b_{22} = K_2 \left(\frac{AE}{L}\right) \tag{34}$$

$$b_{33} = K_3 \left(\frac{EI}{T}\right) \tag{35}$$

$$b_{13} = K_5 \left(\frac{EI}{T^2}\right) \tag{36}$$

$$b_{31} = K_6 \left(\frac{EI}{T^2}\right) \tag{37}$$

Fixity coefficients

34. The constants K_1 through K_6 depend on such variables as the pile head fixity and the distribution of load from the pile to the soil axially and torsionally. Values of K_1 through K_6 can be derived for various degrees of fixity.

35. Knowing the degree of fixity, the following values of K_1 through K_6 can be derived for a lateral subgrade modulus that varies linearly with depth:

Degree of Fixity (DF)	Fixity Coefficients for Linear Subgrade Modulus					
	K_1	K_2	K_3	K_4	K_5	K_6
1.0	1.0756	1.0 for bear-	1.4988	Torsion (as-	0.9990	0.9990
0.9	0.9263	ing or 2.0 for fric-	1.3489	sumed 0.0 by some	0.8991	0.7736
0.8	0.8129	tion piles	1.1990	designers)	0.7992	0.6035
0.7	0.7242	in compres- sion. For	1.0491		0.6993	0.4704
0.6	0.6530	piles in	0.8993		0.5994	0.3636
0.5	0.5945	tension the value	0.7494		0.4995	0.2759
0.4	0.5457	should be	0.5995		0.3996	0.2025
0.3	0.5042	reduced. Suggest 1/2	0.4496		0.2997	0.1404
0.2	0.4687	of value	0.2998		0.1998	0.0870
0.1	0.4378	for com- pression	0.1499		0.0999	0.0406
0.0	0.4107	piles.	0.0		0.0	0.0

36. The value of DF , degree of fixity of a pile into the cap (expressed as a fraction), must be selected with a full understanding of the conditions that must be met for a pile, which is assumed to be fixed, to actually be fixed.

37. The fixity of the pile, DF , depends to a great extent on the pile's embedment into the pile cap. A pretensioned prestressed concrete pile is not fully fixed unless the extension of the pile concrete into the cap is at least as long as the bond development length of the prestressing strands. Further, the pile cannot develop the full moment capacity at the bottom of the cap. Any strand extension distance beyond the end of the pile does not contribute to the bond development distance because the strand elongation needed to develop the strand prestress will cause excessive cracking and loss of rigidity of the concrete. However, a posttensioned concrete pile can be considered fully fixed with less embedment than a pretensioned pile if the tendon(s) are tensioned to the cap after the cap is placed. A nonprestressed concrete pile may be considered fully fixed by a bar extension equal to the bond development length.

Orientation of the
<u>pile to the foundation</u>

38. In a three-dimensional system the pile may be located at a position rotated to the foundation axis and may be battered. Its position in the pile cap is fully defined by the clockwise angle α_i to the direction of batter and the batter slope h_i , as shown in Figure 3. The major principal axis of a pile i , where $I_1 \neq I_2$, should coincide with the angle α_i . The components of force and displacement of the rotated pile axis to the foundation axis are found by the transformation matrix $\{a\}_i$ for pile i where

h_i = batter (h_i Vertical on 1 Horizontal)

α_i = clockwise angle to the batter and/or major principal axis

γ_i = arc cot h_i

In a three-dimensional system

$$\{a'\}_i = \begin{Bmatrix} (\cos\gamma \quad \cos\alpha) & -\sin\alpha & (\sin\gamma \quad \cos\alpha) \\ (\cos\gamma \quad \sin\alpha) & \cos\alpha & (\sin\gamma \quad \sin\alpha) \\ -\sin\gamma & 0 & \cos\gamma \end{Bmatrix} \tag{38}$$

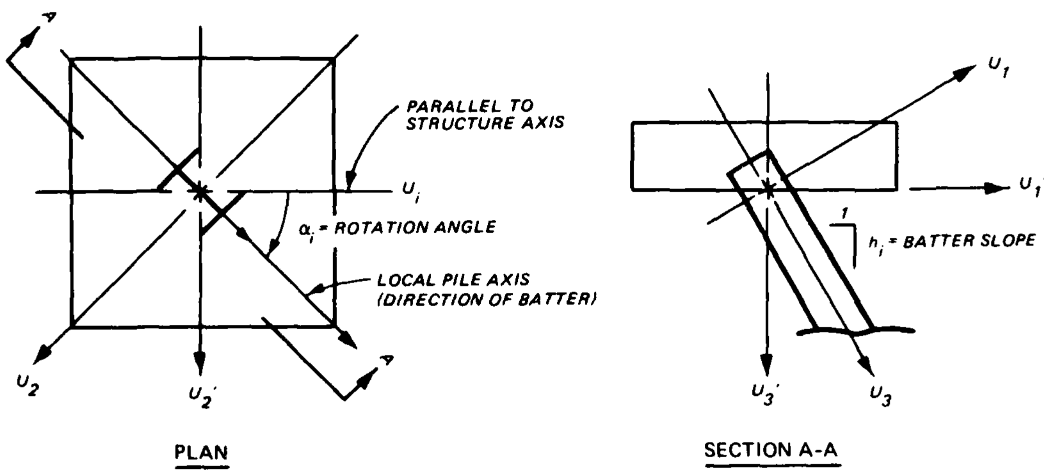

Figure 3. Orientation of local pile axis and global foundation axis

and

$$\{a\}_i = \begin{Bmatrix} a' & 0 \\ 0 & a' \end{Bmatrix} \tag{39}$$

In a two-dimensional system

$$\{a\}_i = \begin{Bmatrix} (\cos\gamma \quad \cos\alpha) & (\sin\gamma \quad \cos\alpha) & 0 \\ -\sin\gamma & \cos\gamma & 0 \\ 0 & 0 & \cos\alpha \end{Bmatrix} \tag{40}$$

39. By the use of the transformation matrix the pile forces can be rotated into forces parallel to the foundation axis by

$$\{F'\}_i = \{a\}_i \{F\}_i \tag{41}$$

and

$$\{x\}_i = \{a\}_i^T \{x'\}_i \tag{42}$$

By substitution

$$\{F'\}_i = \{a\}_i \{b\}_i \{a\}_i^T \{x'\}_i \tag{43}$$

which is the relationship of the pile forces to their deflections in an orthogonal coordinate system parallel to the foundation axes.

Coordinate location of the pile in the foundation

40. Pile i may be located in the foundation with axes through its origin parallel to the foundation axes. The foundation loads $\{Q\}$ and displacements $\{\Delta\}$ are located with respect to the foundation axes.

41. The forces $\{F'\}_i$ due to the pile on the pile cap are in equilibrium with a set of forces $\{q\}_i$ at the coordinate center of the pile cap.

Equilibrium yields

$$\{q\}_i = \{c\}_i\{F'\}_i \tag{44}$$

in which $\{c\}_i$, the statics matrix for a three-dimensional system, is

$$\{c\}_i = \begin{Bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -u_3 & u_2 & 1 & 0 & 0 \\ u_3 & 0 & -u_1 & 0 & 1 & 0 \\ -u_2 & u_1 & 0 & 0 & 0 & 1 \end{Bmatrix} \tag{45}$$

The statics matrix $\{c\}_i$ for a two-dimensional system is

$$\{c\}_i = \begin{Bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ u_3 & -u_1 & 1 \end{Bmatrix} \tag{46}$$

where

$u_1 = U_1$ coordinate of the pile, LENGTH
$u_2 = U_2$ coordinate of the pile, LENGTH
$u_3 = U_3$ coordinate of the pile, LENGTH

Foundation stiffness analysis

42. If the pile cap is assumed rigid, then the deflection of the pile cap can be related to the deflection of the piling in the foundation axis coordinates by

$$\{x'\}_i = \{c\}_i^T\{\Delta\} \tag{47}$$

43. The foundation load $\{Q\}$ is distributed to each pile so that

$$\{Q\} = \sum_{i=1}^{n} \{q\}_i \tag{48}$$

where n = number of piles. The relationships between the foundation load and the pile cap deflections are

$$\{Q\} = \{S\}\{\Delta\} \tag{49}$$

in which $\{S\}$ is the stiffness influence coefficients matrix for the foundation as a whole. The $\{S\}$ matrix is found by introducing the contribution of each individual pile toward the stiffness of the pile cap. This yields

$$\{q\}_i = \{S'\}_i\{\Delta\} \tag{50}$$

in which

$$\{S'\}_i = \{c\}_i\{a\}_i\{b\}_i\{a\}_i^T\{c\}_i^T \tag{51}$$

and finally

$$\{S\} = \sum_{i=1}^{n} \{S'\}_i \tag{52}$$

Once the stiffness matrix is known for the total foundation, the problem is essentially solved and only requires back substitution to find the distribution of loads to the individual pile. It can be noted that the foundation stiffness matrix $\{S\}$ is independent of the external loads.

Loads and displacements

44. The displacements of the pile cap can be found by inverting the foundation stiffness matrix $\{S\}$ and multiplying it by the

external load matrix $\{Q\}$ or

$$\{\Delta\} = \{S\}^{-1}\{Q\} \tag{53}$$

Once the foundation deflections are known the deflection of pile i about its own axes can be found by

$$\{x\}_i = \{a\}_i^T\{c\}_i^T\{\Delta\} \tag{54}$$

Finally, the forces allotted to each pile about its axes can be found from Equation 1 where

$$\{F\}_i = \{b\}_i\{x\}_i \tag{55}$$

It may be desirable to resolve the forces along the pile axes to forces parallel to the structure coordinate axes. These can be found by

$$\{F'\}_i = \{a\}_i\{b\}_i\{a\}_i^T\{c\}_i^T\{\Delta\} \tag{56}$$

Failure Criteria

Allowable loads

45. The allowable axial loads for combined bending (ACB), the allowable moment about the minor principal axis (AMIN), and the allowable moment about the major principal axis (AMAJ) differ in prestressed concrete piles depending on whether the pile is in tension or compression. Therefore, the program allows the user to input two sets of values for the above-mentioned variables, one set for piles in tension and one set for piles in compression. The program checks whether the value of the axial force in the pile is positive (compression) or negative (tension) to determine which set of allowables will be used

for checking failure. The program also allows the user to input an allowable compressive load and an allowable tensile load.

Combined bending factor

46. The combined bending factor for a three-dimensional case is computed as (a) the absolute value of the vertical pile force divided by the allowable axial load plus (b) the absolute value of the moment about the U_1 axis divided by the allowable moment about the minor axis plus (c) the absolute value of the moment about the U_2 axis divided by the allowable moment about the major axis. The pile is considered to fail if the combined bending factor is greater than one.

Buckling

47. The program calculates a buckling factor for a constant soil modulus or a linearly varying soil modulus. For a constant soil modulus the buckling factor is

$$\mathrm{PBUCK} = (7 \times \mathrm{DF} \times \frac{(1 + \mathrm{PR})}{56.0} \times \mathrm{E} \times \mathrm{AMIN1}\left((I_1 \times E_s),\ (I_2 \times E_s)\right) \tag{57}$$

where

DF = degree of fixity

PR = pile resistance (end bearing or friction)

E = modulus of elasticity of pile material

AMIN1 = minimum of two values in parentheses

X = pile dimension parallel to U_1-axis of the pile

48. For a linearly varying soil modulus the buckling factor is

$$\mathrm{PBUCK} = \left(7 \times \mathrm{DF} \times \frac{(1 + \mathrm{PR})}{56.0} \times 1.57 \times \frac{E_s}{X}\right)^{2.0} \times E^{3.0} \times \mathrm{AMIN1}\left(X^{2.0} \times I_1^{\,3.0}\ ,\ X^{2.0} \times I_2^{\,3.0}\right)^{0.2} \tag{58}$$

49. A pile fails in buckling if the buckling factor, PBUCK , is greater than zero and less than the axial force in the pile.

Compression and tension

50. If a pile is in compression, it fails when the allowable compressive load is exceeded by the axial force in the pile. If a pile is in tension, it fails when the allowable tensile load is exceeded by the absolute value of the axial force in the pile.

PART IV: USER'S GUIDE FOR PROGRAM LMVDPILE

General Introduction

51. Documentation for the computer program LMVDPILE (analysis of two- and three-dimensional pile foundations) is presented herein and includes a general introduction, program listing, flow charts, guide for data input, and input-output data for several example problems.

52. LMVDPILE is a general direct stiffness analysis computer program that can be used to determine structure deflections, pile deflections, and forces acting on a group of piles placed in soil and topped with a rigid cap.

53. In the analysis used in LMVDPILE, the base (pile cap) is assumed to be rigid, and the structure and soil are considered to behave in a linear-elastic manner. Each pile behavior in a three-dimensional problem is represented by a 6 by 6 stiffness matrix and in a two-dimensional problem by a 3 by 3 stiffness matrix (Hrennikoff 1950, Saul 1968). The elastic pile constants b_{ij} are dependent on many factors, as shown in Part III, and can be obtained by using the sets of equations given. The direct stiffness method is then used to analyze the problem.

54. Two companion programs are available for use with LMVDPILE. One is a preprocessor routine (PILESTF) which will calculate the pile-head stiffness matrix b_{ij} for a pile in layered soil with a lateral subgrade modulus E_s varying with depth as follows:

$$E_s = K_1 + K_2 z^n \qquad (59)$$

where

z = depth

K_i, K_2, n = soil parameters

When K_2 equals zero, E_s is a constant (such as for clays). When K_1 equals zero and n equals 1.0, E_s is linearly varying (such as for sands). The pile-head stiffness can be used as input to the LMVD-PILE program. Documentation for PILESTF is presented in Appendix A.

55. The second program is an interactive graphics postprocessor display program (FDRAW). Program LMVDPILE writes an output file which is used by FDRAW to display geometry, batter, pile loads, and load factors as calculated by program LMVDPILE. Documentation for FDRAW is presented in Appendix B. A pile optimization program that can help in designing pile layouts is also being developed.

56. LMVDPILE can be run on the WES G-635, Macon H6000, and Boeing CDC computers in the time-sharing mode. The program is part of the CORPS (Conversationally Oriented Real-Time Program-Generating System) library. It is identified by the program number X0034. To execute the program, issue the appropriate run command given below:

a. On the WES or Macon computer

```
RUN WESLIB/CORPS/X0034,R
```

b. On the Boeing computer

```
OLD,CORPS/UN=CECELB
CALL,CORPS,X0034
```

Data may be input interactively at execute time or may be input as a prepared data file. Output may be directed to an output file or come directly back to the terminal.

Flow Charts

57. A flow chart for the program is shown in Figure 4. The sequence of operations for subroutine BMAT, a subroutine to calculate elastic pile constants, is diagrammed in Figure 5.

Data Input for LMVDPILE

58. Data input to program LMVDPILE is basically the same for a two- or three-dimensional analysis. However, for the user's convenience, the data input guide for a two-dimensional analysis is given first. Then the data input guide for a three-dimensional analysis is given.

Guide for two-dimensional data input

59. Data for a two-dimensional analysis should be input to program

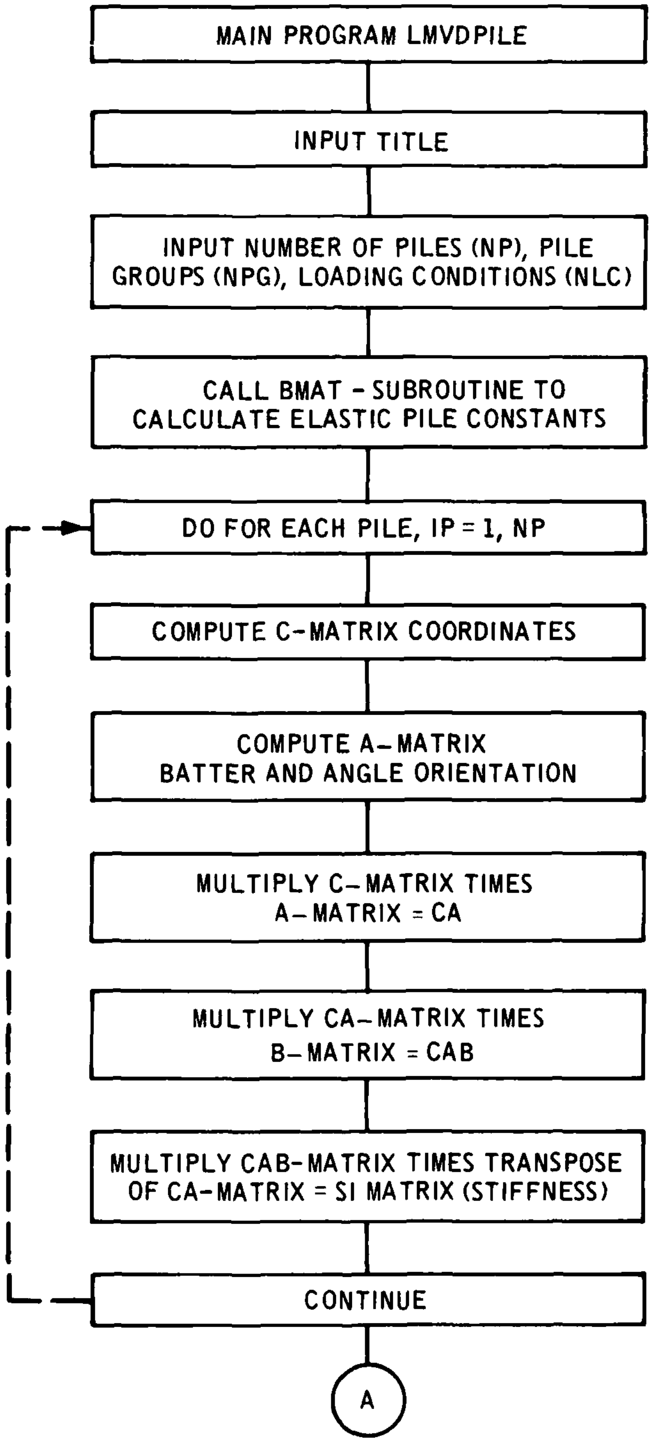

Figure 4. Flow chart for LMVDPILE (sheet 1 of 2)

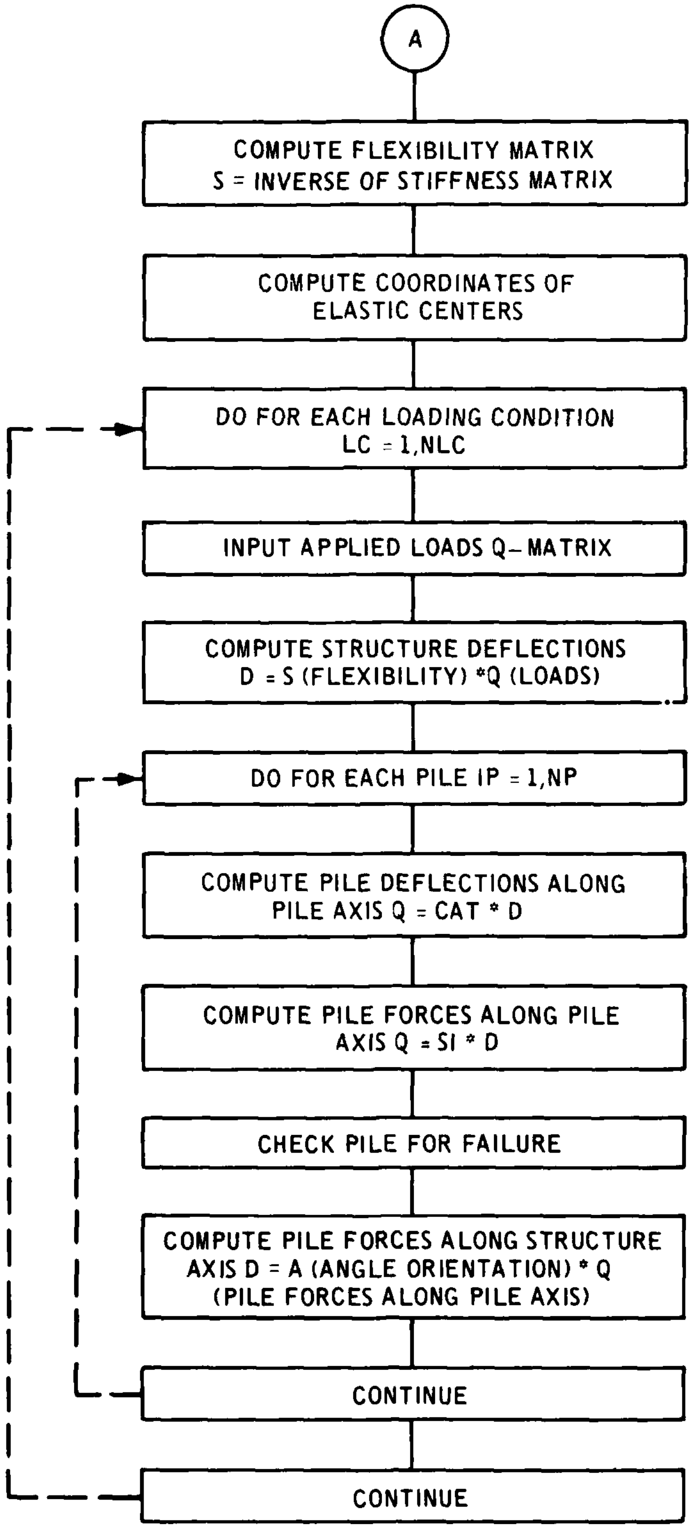

Figure 4 (sheet 2 of 2)

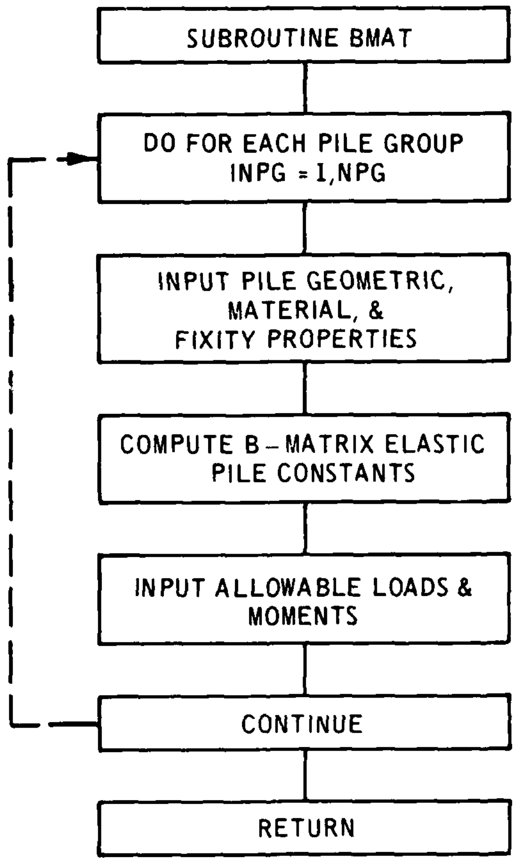

Figure 5. Flow chart of subroutine BMAT for LMVDPILE

LMVDPILE according to the following guide. All input is in free field (a comma or at least one blank should separate data items). Data can be input either interactively or from a data file. If a data file is created, use line numbers for each data line.

Group 1 - Title

A. | TITLE |

TITLE = 66-character problem heading

B. | TITLE1 |

TITLE1 = second 66-character problem heading

Group 2 - Control Data for Piles and Loads

A. | ITYPE |

ITYPE = code for type of analysis
2 --- two dimensional

B. | NP, NPG, NLC |

NP = number of pile rows

NPG = number of pile groups
NLC = number of loading conditions

Group 3 - Control and Data for Soil Properties

MV, ES

MV = type of soil modulus variance
- 1 --- constant soil modulus
- 2 --- linearly varying soil modulus

ES = subgrade modulus (units are in psi) for MV = 1
ES = KS = coefficient of subgrade modulus (pci) for MV = 2

Group 4 - Control and Data for Elastic Pile Constants

Note: Groups 4-8 should be repeated NPG (number of pile groups) number of times.

A.

NPA, NPB, SLEN, NPS

NPA = identification number of first pile in pile group
NPB = identification number of last pile in pile group
SLEN = length of pile (feet)
NPS = code for type of input to compute elastic pile constants (B-matrix terms)
- 1 --- input B-matrix terms directly
- 2 --- any shape pile
- 3 --- round pile

B. Note: Necessary only if NPS = 1

B11, B22, B33, B31

BIJ = elastic pile constant for row I, column J

C. Note: Necessary only if NPS = 2

AIX, AIY, AREA, X, Y

AIX = I_1, moment of inertia about local U_1 axis (in.^{4})
AIY = I_2, moment of inertia about local U_2 axis (in.^{4})
AREA = cross-sectional area of pile (in.^{2})
X = pile dimension parallel to U_1 axis (in.)
Y = pile dimension parallel to U_2 axis (in.)

D. Note: Necessary only if NPS = 3

D

D = average diameter of piles in the groups (in.)

Group 5 - Control and Data for Pile Material

A.

MP

MP = type of material
- 1 --- concrete (E calculated from US input in item 5B)

2 --- timber (E set to 1,760,000 psi*)
3 --- steel (E set to 29,000,000 psi)
4 --- special (E input in item 5C)

B. Note: Necessary only if MP = 1

US , W

US = ultimate strength of concrete (psi)
W = weight of concrete (pcf)

C. Note: Necessary only if MP = 4

E

E = modulus of elasticity (psi)

Group 6 - Control and Data for Fixity Coefficients to Describe Pile

A.

NF

NF = code for input of fixities
1 --- input degree of fixity and all coefficients
2 --- input degree of fixity

B. Note: Necessary only if NF = 1. See paragraph 35.

DF, K1, K2, K3, K4, K5, K6

DF = degree of fixity of pile head-to-base (values between 0 and 1)
K1 = lateral fixity coefficient
K2 = pile axial resistance coefficient
1.0 --- end bearing pile in compression
2.0 --- friction pile in compression

For piles in tension the value should be reduced. Suggest one half of value for compression piles.

K3 = rotational fixity coefficient
K4 = coefficient for torsion
K5 = fixity constant
K6 = fixity constant

C. Note: Necessary only if NF = 2

DF, PR, PFT, G

DF = degree of fixity of pile head-to-base (one of the three values given below)
0.0 --- hinged pile head
0.5 --- partially fixed pile head
1.0 --- fixed pile head

---

* A table of factors for converting U. S. customary units of measurement to metric (SI) units is presented on page 4.

PR = pile axial resistance coefficient, K_2
  1.0 --- end bearing pile in compression
  2.0 --- friction pile in compression

  For piles in tension the value should be reduced. Suggest one half of value for compression piles.

PFT = participation factor for torsion, K_4 (values between 0 and 1) (equals zero for 2-D problem)
G = torsion modulus (psi) (equals zero for 2-D problem)

Group 7 - Data for 2-D Analysis (ITYPE = 2)

NROW

NROW = number of similar rows

Group 8 - Data for Allowable Pile Loads and Moments

ACL, ATL, ACB, AMAJ

ACL = allowable compressive load (kips)
ATL = allowable tensile load (kips)
ACB = allowable compressive load in bending (kips)
AMAJ = allowable moment (kip-ft)

Note: Repeat groups 4-8 data NPG (number of pile groups) number of times.

Group 9 - Control and Data for Pile Orientation

A.

IB

IB = code for input of batter and angle
  0 --- input batter and angle for each pile
  >0 --- number of subgroups of piles in the group with the same batter and angle orientation

B. Note: Necessary only if IB (number of subgroups) > 0 . Repeat IB number of times.

NFP, NLP, BATT

NFP = identification number of first pile in subgroup
NLP = identification number of last pile in subgroup
BATT = batter "BATT" vertical on 1 horizontal
  <0 --- pile slopes from top right to lower left
  =0 --- vertical pile
  >0 --- pile slopes from top left to lower right

Group 10 - Pile Data for 2-D Pile Groups (ITYPE=2)

Note: Necessary only if IB > 0

U1(1) , U1(2) , U1(3) ,... U1(NP)

* U1 = distance from origin to pile along U_1-axis.

Group 11 - Data for Pile Orientation

Note: Necessary only if IB = 0 and ITYPE = 2 (2-D pile groups). Repeat NP (number of pile rows) number of times

H, U1

H = batter H vertical on 1 horizontal
- <0 --- pile slopes from top right to lower left
- 0 --- vertical pile
- >0 --- pile slopes from top left to lower right

U1 = distance from origin to pile along U_1 axis (feet)

Group 12 - Data for applied Loads and Moments

Note: Repeat NLC (number of loading conditions) number of times.

Q1, Q3, Q5

Q1 = horizontal load along U_1 axis (kips)
Q3 = vertical load along U_3 axis (kips)
Q5 = moment about U_2 axis (kip-ft)

Guide for three-dimensional data input

58. Data for a three-dimensional analysis should be input to program LMVDPILE according to the following guide. All input is in free-field (a comma or at least one blank should separate data items). Data can be input either interactively or from a data file. If a data file is created, use line numbers for each data line.

Group 1 - Title

A.
TITLE

TITLE = 66-character problem heading

B.
TITLE1

TITLE1 = second 66-character problem heading

Group 2 - Control Data for Piles and Loads

A.
ITYPE

---

* Successive piles with the same coordinate may be input in the form:

N * U

where N = the number of piles with the same coordinates
U = the value of the coordinate in feet

ITYPE = code for type of analysis

3 --- three dimensional

B. NP, NPG, NLC

NP = total number of piles
NPG = number of pile groups
NLC = number of loading conditions

Group 3 - Control and Data for Soil Properties

MV, ES

MV = type of soil modulus variance
1 --- constant soil modulus
2 --- linearly varying soil modulus
ES = subgrade modulus (psi) for MV = 1
ES = KS = coefficient of subgrade modulus (pci) for MV = 2

Group 4 - Control and Data for Elastic Pile Constants

Note: Groups 4-7 should be repeated NPG (number of pile groups) number of times.

A. NPA, NPB, SLEN, NPS

NPA = identification number of first pile in pile group
NPB = identification number of last pile in pile group
SLEN = length of pile (feet)
NPS = code for type of input to compute elastic pile constants (B-matrix terms)
1 --- input B-matrix terms directly
2 --- any shape pile
3 --- round pile

B. Note: Necessary only if NPS = 1

B11, B22, B33, B44, B55, B66, B42, B51

B11, etc = elastic pile constants

C. Note: Necessary only if NPS = 2

AIX, AIY, AREA, X, Y

AIX = I_1, moment of inertia about local U_1 axis (in.^{4})
AIY = I_2, moment of inertia about local U_2 axis (in.^{4})
AREA = cross-sectional area of pile (in.^{2})
X = pile dimension parallel to U_1 axis (in.)
Y = pile dimension parallel to U_2 axis (in.)

D. Note: Necessary only if NPS = 3

D

D = average diameter of piles in the groups (in.)

Group 5 - Control and Data for Pile Material

A. | MP |
|---|

MP = type of material
- 1 --- concrete (E calculated from US input in item 5B)
- 2 --- timber (E set to 1,760,000 psi)
- 3 --- steel (E set to 29,000,000 psi)
- 4 --- special (E input in item 5C)

B. Note: Necessary only if MP = 1

US , W

US = ultimate strength of concrete (psi)
W = weight of concrete (pcf)

C. Note: Necessary only if MP = 4

E

E = modulus of elasticity (psi)

Group 6 - Control and Data for Fixity Coefficients to Describe Pile

A. | NF |
|---|

NF = code for input of fixities
- 1 --- input degree of fixity and all coefficients
- 2 --- input degree of fixity

B. Note: Necessary only if NF = 1. See paragraph 35.

DF, K1, K2, K3, K4, K5, K6

DF = degree of fixity of pile head-to-base (values between 0 and 1)
K1 = lateral fixity coefficient
K2 = pile axial resistance coefficient
- 1.0 --- end bearing pile in compression
- 2.0 --- friction pile in compression

For piles in tension the value should be reduced. Suggest one half of value for compression piles.

K3 = rotational fixity coefficient
K4 = coefficient for torsion
K5 = fixity constant
K6 = fixity constant

C. Note: Necessary only if NF = 2

DF, PR, PFT, G

DF = degree of fixity of pile head-to-base (one of the three values given below)
- 0.0 --- hinged pile head
- 0.5 --- partially fixed pile head
- 1.0 --- fixed pile head

PR = pile axial resistance coefficient, K_2
  1.0 --- end bearing pile in compression
  2.0 --- friction pile in compression

  For piles in tension the value should be reduced. Suggest one half of value for compression piles.

PFT = participation factor for torsion (values between 0 and 1), K_4
G = torsion modulus (psi)

Group 7 - Data for Allowable Pile Loads and Moments

ACBT, AMINT, AMAJT, ACBC, AMINC, AMAJC, ACL, ATL

ACBT = allowable axial load used in combined bending equation for pile in tension (kips)
AMINT = allowable moment about minor principal axis for pile in tension (kip-ft)
AMAJT = allowable moment about major principal axis for pile in tension (kip-ft)
ACBC = allowable axial load used in combined bending equation for pile in compression (kips)
AMINC = allowable moment about minor principal axis for pile in compression (kip-ft)
AMAJC = allowable moment about major principal axis for pile in compression (kip-ft)
ACL = allowable compressive load (kips)
ATL = allowable tensile load (kips)

Note: Repeat groups 4-7 data NPG (number of pile groups) number of times

Group 8 - Control and Data for Pile Orientation

A.

IB

IB = code for input of batter and angle
  0 --- input batter and angle for each pile
  >0 --- number of subgroups of piles in the group with the same batter and angle orientation

B. Note: Necessary only if IB (number of subgroups) > 0 .

NFP, NLP, BATT, ANGL

NFP = identification number of first pile in subgroup
NLP = identification number of last pile in subgroup
BATT = batter "BATT" vertical on 1 horizontal
  0 --- vertical pile
ANGL = clockwise angle between the positive U_1 axis of the structure and the U_1 axis (direction of batter) of the pile (degrees)

Group 9 - Pile Data for 3-D Pile Groups (ITYPE = 3)

Note: Necessary only if IB > 0.

A. | U1(1), U1(2), U1(3),... U1(NP) |

* U1 = distance from origin to pile along U_1 axis (feet)

B. | U2(1), U2(2), U2(3),... U2(NP) |

* U2 = distance from origin to pile along U_2 axis (feet)

C. | U3(1), U3(2), U3(3),... U3(NP) |

* U3 = distance from origin to pile along U_3 axis (feet)

Group 10 - Data for Pile Orientation 3-D Pile Group (ITYPE = 3)

Note: Necessary only if IB = 0. Repeat NP (number of piles) number of times.

| H, ANG, U1, U2, U3 |

H = batter H vertical on 1 horizontal
0 --- vertical pile
ANG = clockwise angle between the positive U_1 axis of the structure and the U_1 axis (direction of batter) of the pile (degrees)
U1 = distance from origin to pile along U_1-axis (feet)
U2 = distance from origin to pile along U_2-axis (feet)
U3 = distance from origin to pile along U_3-axis (feet)

Group 11 - Data for Applied Loads and Moments

Note: Repeat NLC (number of loading conditions) number of times.

| Q1, Q2, Q3, Q4, Q5, Q6 |

Q1 = horizontal load along U_1 axis (kips)
Q2 = horizontal load along U_2 axis (kips)
Q3 = vertical load along U_3 axis (kips)
Q4 = moment about U_1 axis (kip-ft)
Q5 = moment about U_2 axis (kip-ft)
Q6 = moment about U_3 axis (kip-ft)

---

* Successive piles with the same coordinate may be input in the form:

N * U

where N = the number of piles with the same coordinates
U = the value of the coordinate in feet

PART V: EXAMPLE PROBLEMS

Example Problem 1

Two-dimensional problem, 2 pinned piles with constant soil modulus

59. This example problem illustrates the use of LMVDPILE for a two-dimensional system supported by four vertical piles. The physical problem is shown in Figure 6. (Example problem 7 is the three-dimensional run of this same problem; Figure 16, page 83, shows the plan view of the system.) Figure 7 shows the properties and loading conditions for this example. Input data are saved in a file and listed in Table 2. The computer output is presented in Table 3.

60. This example serves as a means to verify the computer output by comparison with manual calculations.

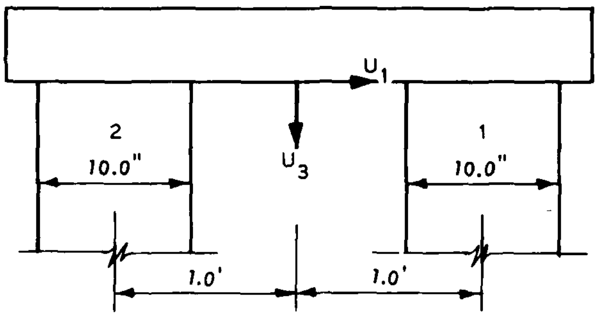

Figure 6. Physical problem for example problem 1

Results and calculations

61. The pile forces can be calculated by satisfying equilibrium $\Sigma F = 0$. These were found to agree with the program output shown in Table 3. For example, in loading case 2 there are two rows of piles each having 2 piles subjected to a 1-kip vertical load. The force on each pile is

$$F_H = 1/4 \text{ (applied vertical load)} = 1/4 \text{ (1 kip)} = 0.25 \text{ kip}$$

The displacement in each pile is equal to

Properties	
Ult. str. of concrete = 5000.0 psi	Vertical (h = 0.0)
ES = 10.0 psi	Degree of fixity = 0.0
I_1 = 833.333 $in.^4$	Pile resistance (K2) = 1.0
I_2 = 833.333 $in.^4$	Participation factor for torsion (K4) = 0.0
Area = 100 $in.^2$	Torsion modulus = 0.0
Length = 100 ft	

Loading Case	Q_1 (kips)	Q_2 (kips)	Q_3 (kip-ft)
1	1.0	0.0	0.0
2	0.0	1.0	0.0
3	0.0	0.0	1.0
4	1.0	1.0	1.0

Figure 7. Properties and loading conditions for example problem 1

Table 1

Interactively Input Data for Example Problem 1

```
INPUT DATA FILE NAME IN 6 CHARACTERS OR LESS. HIT A
CARRIAGE RETURN IF INPUT DATA WILL COME FROM TERMINAL.
    ?



INPUT A FILE NAME FOR DATA.  HIT A CARRIAGE RETURN
IF YOU DO NOT WANT TO SAVE DATA FILE.
    ? DATA1


INPUT TWO LINES OF PROJECT IDENTIFICATION NOT
TO EXCEED 66 CHARACTERS EACH

INPUT FIRST LINE
    ? EXAMPLE PROBLEM NO. 1
INPUT SECOND LINE
    ? VERTICAL PILES WITH UNIT LOADS


DO YOU WANT TO RUN A 2-D OR 3-D ANALYSIS?
ENTER 2 OR 3 ? 2


INPUT TOTAL NUMBER OF PILE ROWS IN FOUNDATION
NUMBER OF PILE GROUPS AND LOADING CONDITIONS
    ? 2,1,4


INPUT SOIL PROPERTY DATA - MV AND ES:
      MV=1=CONSTANT SOIL OR MV=2=LINEARLY VARYING SOIL
      ES=SUBGRADE MODULUS (PSI IF MV=1 OR PCI IF MV=2)
    ? 1,10.0


DATA FOR PILE GROUP NO. -    1


INPUT PILE SHAPE DATA:
      NPA=IDENTIFICATION NUMBER OF FIRST PILE ROW IN GROUP
      NPB=IDENTIFICATION NUMBER OF LAST PILE ROW IN GROUP
      SLEN=LENGTH OF PILES (FEET)
      NPS=CODE FOR TYPE OF INPUT TO COMPUTE ELASTIC PILE CONSTANTS
          1=INPUT PILE B MATRIX TERMS DIRECTLY
          2=ANY SHAPE PILE
          3=ROUND PILE

    ? 1,2,100.0,2


INPUT AIX & AIY-MOMENTS OF INERTIA (IN**4)
      AREA - CROSS SECTIONAL AREA (IN**2)
      X & Y - PILE DIMENSIONS ALONG X & Y AXES (INCHES)
    ? 833.333,833.333,100.0,10.0,10.0


INPUT PILE MATERIAL DATA-MP (1=CONCRETE, 2=TIMBER, 3=STEEL, 4=SPECIAL)
    ? 1


INPUT US=ULTIMATE STRENGTH OF CONCRETE (PSI)
      W=WEIGHT OF CONCRETE (PCF)
    ? 5000.0,150.0


INPUT FIXITY DATA - NF (1=INPUT ALL FIXITY COEFFICIENTS
                        OR 2=INPUT DEGREE OF FIXITY
    ? 2


INPUT DF - DEGREE OF FIXITY (0.0,0.5,1.0)
      PP - PILE RESISTANCE (1=BEARING OR 0.5=FRICTION
      PFT - PARTICIPATION FACTOR FOR TORSION
      G - TORSION MODULUS (PSI)
    ? 0.0,1.0,0.0,0.0
```

(Continued)

Table 1 (Concluded)

```
INPUT NUMBER OF SIMILAR ROWS IN GROUP   1 ? 2

INPUT ALLOWABLE LOADS:
      ACL = ALLOWABLE COMPRESSIVE LOAD (KIPS)
      ATL = ALLOWABLE TENSILE LOAD (KIPS)
      ACB = ALLOWABLE COMPRESSIVE LOAD IN BENDING (KIPS)
      AMAJ = ALLOWABLE MOMENT (KIP-FT)
    ? 100.0,100.0,100.0,100.0

INPUT IB: 0=INPUT BATTER FOR EACH PILE OR
      THE NUMBER OF SUBGROUPS WITH THE SAME BATTER
    ? 0

INPUT PILE ORIENTATION DATA
      H-BATTER=H VERTICAL ON 1 HORIZONTAL
      POSITIVE IF BATTERED  TO RIGHT & NEGATIVE IF TO LEFT
      U1=DISTANCE FROM ORIGIN TO PILE ROW(FEET)
   1    ? 0.0,1.0
   2    ? 0.0,-1.0

INPUT APPLIED LOADS AND MOMENT:
      Q1-HORIZONTAL LOAD ALONG U1-AXIS (KIPS)
      Q3-VERTICAL LOAD ALONG U3-AXIS (KIPS)
      Q5-MOMENT ABOUT U2-AXIS (KIP-FEET)

FOR LOADING CONDITION -   1 ? 1.0,0,0

FOR LOADING CONDITION -   2 ? 0,1.0,0

FOR LOADING CONDITION -   3 ? 0,0,1.0

FOR LOADING CONDITION -   4 ? 1.0,1.0,1.0

THIS PROGRAM GENERATES THE FOLLOWING TABLES:

TABLE NO.                  CONTENTS
   1                 PILE AND SOIL DATA
   2                 PILE COORDINATES AND BATTER
   3                 STIFFNESS AND FLEXIBILITY MATRICES FOR THE
                     STRUCTURE AND COORDINATES OF ELASTIC CENTER
   4                 APPLIED LOADS
   5                 STRUCTURE DEFLECTIONS
   6                 PILE DEFLECTIONS ALONG PILE AXIS
   7                 PILE FORCES ALONG PILE AXIS
   8                 PILE FORCES ALONG STRUCTURE AXIS

INPUT THE NUMBERS OF THE TABLES FOR WHICH YOU WANT THE OUTPUT.
SEPARATE THE NUMBERS WITH COMMAS. ? 1,2,3,4,5,6,7,8

INPUT A FILENAME FOR TABLE 8 IN 8 CHARACTERS OR LESS
IF YOU WANT TO USE THIS INFORMATION FOR A NEW RUN
HIT A CARRIAGE RETURN IF YOU DO NOT WANT THIS FILE.
    ?

INPUT A FILE NAME FOR OUTPUT IN 8 CHARACTERS OR LESS.
HIT A CARRIAGE RETURN IF OUTPUT IS TO BE PRINTED ON TERMINAL.
    ?

INPUT A FILE NAME IN 8 CHARACTERS OR LESS FOR PLOT DATA NECESSARY
FOR PROGRAM FDRAW. HIT A CARRIAGE RETURN IF YOU DO NOT WANT TO
SAVE THIS FILE
    ?
```

Table 2

Input Data for Example Problem 1

Group								
1A	10000	EXAMPLE PROBLEM NO. 1						
1B	10010	VERTICAL PILES WITH UNIT LOADS						TITLE
2A	10020	2						2-D ANALYSIS
2B	10030	2	1	4				NUMBER OF PILES, PILE GROUPS, LOADING CONDITIONS
3A	10040	1	10.000					SOIL PROPERTIES
4A	10050	1	2	100.000	2			PILE GEOMETRY
4C	10060	833.333		833.333	100.000	10.000	10.000	
5A	10070	1						
5B	10080	5000.000		150.000				PILE MATERIAL
6A	10090	2						
6C	10100	0.		1.000	0.	0.		PILE FIXITY
7	10110		2					NUMBER OF ROWS
8	10120	100.000		100.000	100.000	100.000		ALLOWABLE LOADS
9A	10130	0						
11	10140	0.		1.000				PILE BATTER AND LOCATION
	10150	0.		-1.000				
12	10160	1.000		0.	0.			
	10170	0.		1.000	0.			APPLIED LOADINGS
	10180	0.		0.	1.000			
	10190	1.000		1.000	1.000			

Table 3

Output Data for Example Problem 1

```
EXAMPLE PROBLEM NO. 1
VERTICAL PILES WITH UNIT LOADS

NO. OF PILE ROWS =   2    B MATRIX IS CALCULATED FOR EACH ROW


************************************************************************


 1. TABLE OF PILE AND SOIL DATA

PILE NUMBERS

   1   2  E = 0.43E 07 PSI    IX =    833.33 IN**4    IY =    833.33 IN**4
          AREA =  100.0 IN**2   X =      10.00 IN      Y =     10.00 IN
          LENGTH =  100.0 FEET    ES =  10.000
          K1 =   0.4107  K2 =   1.0000  K3 =   0.
          K4 =   0.      K5 =   0.      K6 =   0.


          ALLOWABLES:  COMPRESSIVE LOAD =     100.000 KIPS
                       TENSILE LOAD =     100.000 KIPS
                       BENDING =    100.000 KIPS
                       MOMENT =    100.000 KIP-FT


 THE B MATRIX FOR PILES   1 THROUGH   2 IS

  0.972E 03  0.         0.
  0.         0.357E 06  0.
  0.         0.         0.


************************************************************************


 2. TABLE OF PILE COORDINATES AND BATTER

PILE ROW  BATTER          U1 (FT)
   1      VERTICAL         1.000
   2      VERTICAL        -1.000


************************************************************************


 3. STIFFNESS MATRIX S FOR THE STRUCTURE

  0.389E 04  0.         0.
  0.         0.143E 07  0.
  0.         0.         0.206E 09

 3A  FLEXIBILITY MATRIX F FOR THE STRUCTURE

  0.257E-03  0.         0.
  0.         0.700E-06  0.
  0.         0.         0.486E-08


COORDINATES OF ELASTIC CENTER
  EC1 =     0.        EC2 =     0.
```

(Continued)

(Sheet 1 of 5)

Table 3 (Continued)

```
******** LOADING CONDITION 1 ********

************************************************************************

4. MATRIX OF APPLIED LOADS Q (KIPS & FEET)

       Q1          Q3          Q5
      1.000       0.           0.

************************************************************************

5. STRUCTURE DEFLECTIONS (INCHES)

    D1          D3          D5
 0.257E 00  0.          0.

************************************************************************

6. PILE DEFLECTIONS ALONG PILE AXIS (INCHES)

 PILE      X1          X3          X5
  1  0.257E 00  0.          0.
  2  0.257E 00  0.          0.

************************************************************************

7. PILE FORCES ALONG PILE AXIS (KIPS & FT)

 PILE   F1       F3       F5           FAILURE
                                       BU CO TE
  1   0.250   0.       0.
  2   0.250   0.       0.

TOTAL NO. FAILURES =  0      LOAD CASE 1

************************************************************************

8. PILE FORCES ALONG STRUCTURE AXIS (KIPS & FEET)

 PILE    F1        F3         F5
  1    0.250     0.         0.
  2    0.250     0.         0.
------------------------------------------------------------------------
SUM    1.000     0.         0.

************************************************************************
```

(Continued)

(Sheet 2 of 5)

Table 3 (Continued)

```
******** LOADING CONDITION 2 ********

************************************************************************

4. MATRIX OF APPLIED LOADS Q (KIPS & FEET)

       Q1          Q3          Q5
      0.          1.000        0.

************************************************************************

5. STRUCTURE DEFLECTIONS (INCHES)

     D1          D3          D5
 0.          0.700E-03  0.

************************************************************************

6. PILE DEFLECTIONS ALONG PILE AXIS (INCHES)

 PILE     X1          X3          X5
  1  0.          0.700E-03  0.
  2  0.          0.700E-03  0.

************************************************************************

7. PILE FORCES ALONG PILE AXIS (KIPS & FT)

 PILE   F1       F3       F5           FAILURE
                                       BU CO TE
  1   0.       0.250    0.
  2   0.       0.250    0.

TOTAL NO. FAILURES =  0      LOAD CASE 2

************************************************************************

8. PILE FORCES ALONG STRUCTURE AXIS (KIPS & FEET)

 PILE     F1        F3        F5
  1      0.        0.250     0.
  2      0.        0.250     0.
------------------------------------------------------------------------
 SUM     0.        1.000    -0.000

************************************************************************
```

(Continued)

(Sheet 3 of 5)

Table 3 (Continued)

```
******** LOADING CONDITION 3 ********


**********************************************************************


4. MATRIX OF APPLIED LOADS Q (KIPS & FEET)

       Q1           Q3           Q5
      0.           0.            1.000


**********************************************************************


5. STRUCTURE DEFLECTIONS (INCHES)

    D1          D3          D5
 0.          0.          0.583E-04


**********************************************************************


6. PILE DEFLECTIONS ALONG PILE AXIS (INCHES)

 PILE      X1          X3          X5
  1  0.          -0.700E-03  0.583E-04
  2  0.           0.700E-03  0.583E-04


**********************************************************************


7. PILE FORCES ALONG PILE AXIS (KIPS & FT)

 PILE    F1       F3        F5            FAILURE
                                          BU CO TE
  1    0.      -0.250     0.
  2    0.       0.250     0.

TOTAL NO. FAILURES =  0       LOAD CASE 3


**********************************************************************


8. PILE FORCES ALONG STRUCTURE AXIS (KIPS & FEET)

 PILE    F1        F3         F5
  1      0.       -0.250      0.
  2      0.        0.250      0.
----------------------------------------------------------------------
SUM      0.        0.         1.000


**********************************************************************
```

(Continued)

(Sheet 4 of 5)

Table 3 (Concluded)

```
******** LOADING CONDITION 4 ********


**********************************************************************


4. MATRIX OF APPLIED LOADS Q (KIPS & FEET)

        Q1          Q3          Q5
      1.000       1.000       1.000


**********************************************************************


5. STRUCTURE DEFLECTIONS (INCHES)

    D1          D3          D5
 0.257E 00  0.700E-03  0.583E-04


**********************************************************************


6. PILE DEFLECTIONS ALONG PILE AXIS (INCHES)

 PILE      X1          X3          X5
  1   0.257E 00 -0.364E-11  0.583E-04
  2   0.257E 00  0.140E-02  0.583E-04


**********************************************************************


7. PILE FORCES ALONG PILE AXIS (KIPS & FT)

 PILE    F1       F3       F5           FAILURE
                                        BU CO TE
  1    0.250   -0.000    0.
  2    0.250    0.500    0.

TOTAL NO. FAILURES =  0      LOAD CASE 4


**********************************************************************


8. PILE FORCES ALONG STRUCTURE AXIS (KIPS & FEET)

 PILE     F1         F3         F5
  1      0.250     -0.000      0.
  2      0.250      0.500      0.
------------------------------------------------------------------------
SUM      1.000      1.000      1.000


**********************************************************************
```

(Sheet 5 of 5)

$$\delta = \frac{PL}{AE} = \frac{\frac{1}{4} \times 1 \times 100 \times 12}{4300 \times 144 \times \frac{100}{144}} = 0.7 \times 10^{-3} \text{ in.}$$

This result also agrees with the computer program results (page 48, item 6).

62. In loading case 3, a 1 kip-ft moment is applied about the U_2-axis. The pile forces can be calculated by satisfying equilibrium $\Sigma M_{U_2} = 0$.

$$\Sigma M_{U_2} = F3_1 \times N \times U1_1 + F3_2 \times N \times U1_2 + Q5$$

where

$F3_m$ = vertical force for pile row m, m = 1,2

N = number of piles in rows

U_1 = distance from origin to pile

$$\therefore \Sigma M_{U_2} = 2F3_1 + 2F3_2 + 1 \text{ kip-ft}$$

From symmetry $F3_1 = F3_2$

$$\therefore |F3| = 0.25 \text{ kip}$$

This result also agrees with the computer program results (page 49, item 8).

63. Load case 4 can be obtained as a superposition of load cases 1 through 3. The deflections of the pile and the load on each pile can be obtained by superimposing the respective results for load cases 1 through 3. The following computations verify these results.

Pile No.	Load Case	Deflections X_1 (in.)	X_3 (in.)	X_5 (rad.)
1	1	0.257	0.	0.
	2	0.	0.7×10^{-3}	0.
	3	0.	-0.7×10^{-3}	0.583×10^{-4}
	4	0.257	0.	0.583×10^{-4} (page 50, item 6)
2	1	0.257	0.	0.
	2	0.	0.7×10^{-3}	0.
	3	0.	0.7×10^{-3}	0.583×10^{-4}
	4	0.257	0.14×10^{-2}	0.583×10^{-4} (page 50, item 6)

Pile No.	Load Case	Loads F_1 (kips)	F_3 (kips)	F_5 (kip-ft)
1	1	0.25	0.	0.
	2	0.	0.25	0.
	3	0.	-0.25	0.
	4	0.25	0.	0. (page 50, item 7)
2	1	0.25	0.	0.
	2	0.	0.25	0.
	3	0.	0.25	0.
	4	0.25	0.50	0. (page 50, item 7)

These results also agree with the computer program results.

Example Problem 2

Two-dimensional prob-
lem, 1 fixed vertical pile

64. This example problem has only one vertical pile completely fixed into the rigid cap. Figure 8 shows the physical problem. (Example problem 8 is the three-dimensional run of this same problem; Figure 19, page 96, shows the plan view for this example.) Figure 9 shows the loading and properties. The input data are stored in a file and are presented in Table 4. The computer output is shown in Table 5.

65. This example is also a means to verify output by comparison with manual calculations and output from example problem 8.

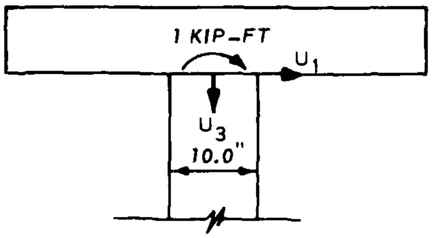

Figure 8. Physical problem for example problem 2

Properties		
Ult. str. of concrete = 5000.0 psi		K_1 = 1.0756
KS = 10.0 pci	DF = 1.0	K_2 = 1.0
I_1 = 833.333 in.^{4}	PR = 1.0	K_3 = 1.4988
I_2 = 833.333 in.^{4}	PFT = 0.0	K_4 = 0.0
Area = 100.0 in.^{2}	G = 0.0	K_5 = 0.9990
Length = 100.0 ft		K_6 = 0.9990
Vertical (h = 0.0)		

Loading Case	Q_1 (kips)	Q_3 (kips)	Q_5 (kip-ft)
1	0.0	0.0	1.0

Figure 9. Properties and loading for example problem 2

Table 4

Input Data for Example Problem 2

Group						
1A	10000	EXAMPLE PROBLEM NO. 2				
1B	10010	ONE FIXED VERTICAL PILE WITH UNIT MOMENT APPLIED				
2A	10020	2				
2B	10030	1	1	1		
3	10040	2	10.000			
4A	10050	1	1	100.000	2	
4C	10060	833.333	833.333	100.000	10.000	10.000
5A	10070	1				
5B	10080	5000.000	150.000			
6A	10090	2				
6C	10100	1.000	1.000	0.	0.	
7	10110		1			
8	10120	100.000	100.000	100.000	100.000	
9A	10130	0				
11	10140	0.	0.			
12	10150	0.	0.	1.000		

Table 5

Output Data for Example Problem 2

```
EXAMPLE PROBLEM NO. 2
ONE FIXED VERTICAL PILE WITH UNIT MOMENT APPLIED

NO. OF PILE ROWS =   1     B MATRIX IS CALCULATED FOR EACH ROW


**********************************************************************


1. TABLE OF PILE AND SOIL DATA

PILE NUMBERS

   1   1  E = 2.43E 07 PSI    IX =    833.33 IN**4    IY =    833.33 IN**4
          AREA =  100.0 IN**2    X =    10.00 IN      Y =    10.00 IN
          LENGTH =  100.0 FEET    ES =  10.000
          K1 =   1.0750  K2 =   1.0000  K3 =   1.4938
          K4 =   0.       K5 =   0.9990  K6 =   0.9990


          ALLOWABLES:  COMPRESSIVE LOAD =    100.000 KIPS
                       TENSILE LOAD =    100.000 KIPS
                       BENDING =    100.000 KIPS
                       MOMENT =    100.000 KIP-FT


 THE B MATRIX FOR PILES    1 THROUGH   1 IS

  0.254E 05  0.         0.135E 07
  0.         0.357E 06  0.
  0.135E 07  0.         0.124E 09


**********************************************************************


 2. TABLE OF PILE COORDINATES AND BATTER

PILE ROW   BATTER          J1 (FT)
   1       VERTICAL        0.


**********************************************************************


 3. STIFFNESS MATRIX S FOR THE STRUCTURE

  0.254E 05  0.         0.135E 07
  0.         0.357E 06  0.
  0.135E 07  0.         0.124E 09


 3A  FLEXIBILITY MATRIX F FOR THE STRUCTURE

  0.926E-04  0.        -0.100E-05
  0.         0.280E-05  0.
 -0.100E-05  0.         0.252E-07


COORDINATES OF ELASTIC CENTER
  EC1 =      0.         EC2 =      0.013
```

(Continued)

Table 5 (Concluded)

```
******** LOADING CONDITION 1 ********


***********************************************************************


4. MATRIX OF APPLIED LOADS Q (KIPS & FEET)

       Q1          Q3          Q5
      Ø.          Ø.           1.ØØØ


***********************************************************************


5. STRUCTURE DEFLECTIONS (INCHES)

     D1         D3          D5
-2.144E-Ø1  Ø.          Ø.3Ø2E-Ø3


***********************************************************************


6. PILE DEFLECTIONS ALONG PILE AXIS (INCHES)

 PILE     X1         X3          X5
  1 -2.144E-Ø1  Ø.         Ø.3Ø2E-Ø3


***********************************************************************


7. PILE FORCES ALONG PILE AXIS (KIPS & FT)

 PILE   F1      F3      F5            FAILURE
                                      BU CC TE
  1   Ø.ØØØ   Ø.      1.ØØØ

TOTAL NO. FAILURES =  Ø      LOAD CASE 1


***********************************************************************


8. PILE FORCES ALONG STRUCTURE AXIS (KIPS & FEET)

 PILE    F1        F3         F5
  1     Ø.ØØØ     Ø.         1.ØØØ
-----------------------------------------------------------------------
SUM     Ø.ØØØ     Ø.         1.ØØØ


***********************************************************************
```

Results and calculations

66. A 1 kip-ft moment was applied about the U_2 axis at the center of the structure where the pile is located. The pile is completely fixed into the rigid cap. Therefore, the resulting moment about the U_2 axis is equal to 1 kip-ft. This result agrees with the computer output shown in Table 5 (item 8).

Example Problem 3

Two-dimensional problem,
Hrennikoff's example
case 2a (very weak soil)

67. This example problem is taken from Hrennikoff's (1950) paper, case 2a. This example is for very weak soil with hinged piles. The physical problem is shown in Figure 10. The properties and loading conditions are shown in Figure 11. The input data are stored in a file prior to the run and are presented in Table 6. The computer output is shown in Table 7.

68. This example serves as a means to verify the computer output with the classical method.

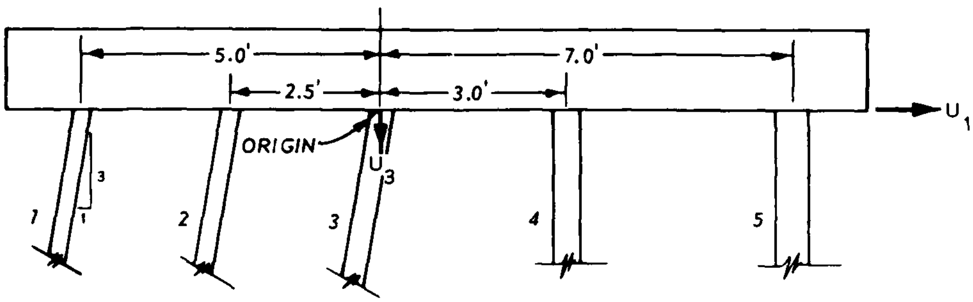

Figure 10. Physical problem for examples 3, 4, and 5

Results and calculations

69. In Hrennikoff's paper manual calculations for this problem are presented. The computer results shown in Table 7 agree closely with his results. A comparison of the two results is presented below. For pile 1,

$$F_1 = 0.442 \text{ kips} \qquad F_3 = 27.395 \text{ kips}$$

as compared with

$$F_1 = 0.44 \text{ kips} \qquad F_3 = 27.5 \text{ kips}$$

Properties	
$E = 0.15 \times 10^7$ psi	Degree of fixity = 0.0
ES = 3.123 psi	Pile resistance (K2) = 0.5
$I_1 = 322.06$ in.^{4}	Participation factor for torsion (K4) = 0.0
$I_2 = 322.06$ in.^{4}	Torsion modulus = 0.0
Area = 63.5 in.^{2}	
Length = 30 ft	

Loading Case	Q_1 (kips)	Q_3 (kips)	Q_5 (kip-ft)
1	-39.375	113.1	173.4

Figure 11. Properties and loading conditions for example problem 3

from case 2a in Hrennikoff's paper. Pile forces along the pile axis for piles 2-5 also agree closely as tabulated below.

	Computer Output		Hrennikoff's Example	
Pile No.	F_1 (kips)	F_3 (kips)	F_1 (kips)	F_3 (kips)
1	0.442	27.395	0.44	27.5
2	0.435	39.282	0.43	39.3
3	0.427	51.170	0.43	51.0
4	0.436	-9.167	0.43	-9.0
5	0.436	10.881	0.43	10.9

Table 6

Input Data for Example Problem 3

Group						
1A	10000	EXAMPLE PROBLEM NO. 3				
1B	10010	HRENNIKOFF'S EXAMPLE - CASE 2A				
2A	10020	2				
2B	10030	5	1	1		
3	10040	1	3.123			
4A	10050	1	5	30.000	3	
4D	10060	9.000				
5A	10070	4				
5C	10080	1500000.000				
6A	10090	2				
6C	10100	0.		0.500	0.	0.
7	10110	1				
8	10120	82.000		40.000	100.000	100.000
9A	10130	2				
9B	10140	1	3	-3.000		
	10150	4	5	0.		
10	10160	-5.000	-2.500	0.	3.000	7.000
12	10170	-39.375		113.1	173.4	

Table 7

Output Data for Example Problem 3

```
EXAMPLE PROBLEM NO. 3
HRENNIKOFF'S EXAMPLE - CASE 2A

NO. OF PILE ROWS =   5     B MATRIX IS CALCULATED FOR EACH ROW


**************************************************************************


1. TABLE OF PILE AND SOIL DATA

PILE NUMBERS

   1    5  E = 0.15E 07 PSI    IX =    322.06 IN**4    IY =    322.06 IN**4
           AREA =   63.6 IN**2    X =      9.00 IN      Y =      9.00 IN
           LENGTH =   30.0 FEET    ES =    3.123
           K1 =   0.4107  K2 =   0.5000  K3 =   0.
           K4 =   0.      K5 =   0.      K6 =   0.

LENGTH OF PILES ( 30.00 FEET) IS INSUFFICIENT
FOR PILE GROUP -   1 MINIMUM ACCEPTABLE LENGTH IS   37.17 FEET
FOR SEMI-INFINITE BEAM ON ELASTIC FOUNDATION

         ALLOWABLES:  COMPRESSIVE LOAD =      82.000 KIPS
                      TENSILE LOAD =     40.000 KIPS

THE B MATRIX FOR PILES   1 THROUGH   5 IS

 0.246E 03  0.         0.
 0.         0.133E 06  0.
 0.         0.         0.


**************************************************************************


2. TABLE OF PILE COORDINATES AND BATTER

PILE ROW  BATTER           U1 (FT)
   1       -3.00          -5.000
   2       -3.00          -2.500
   3       -3.00           0.
   4      VERTICAL         3.000
   5      VERTICAL         7.000


**************************************************************************


3. STIFFNESS MATRIX S FOR THE STRUCTURE

 0.409E 05 -0.119E 06 -0.357E 07
-0.119E 06  0.623E 06 -0.517E 07
-0.357E 07 -0.517E 07  0.164E 10

3A  FLEXIBILITY MATRIX F FOR THE STRUCTURE

 0.193E-03  0.414E-04  0.550E-06
 0.414E-04  0.105E-04  0.123E-06
 0.550E-06  0.123E-06  0.219E-08


COORDINATES OF ELASTIC CENTER
 EC1 =      0.003      EC2 =    -0.002
```

(Continued)

Table 7 (Concluded)

```
******** LOADING CONDITION 1 ********

***************************************************************************

4. MATRIX OF APPLIED LOADS Q (KIPS & FEET)

      Q1         Q3         Q5
   -39.375    113.100    173.400

***************************************************************************

5. STRUCTURE DEFLECTIONS (INCHES)

    D1          D3          D5
-0.177E 01 -0.183E 00 -0.315E-02

***************************************************************************

6. PILE DEFLECTIONS ALONG PILE AXIS (INCHES)

 PILE     X1          X3          X5
  1 -0.180E 01  0.207E 00 -0.315E-02
  2 -0.177E 01  0.296E 00 -0.315E-02
  3 -0.174E 01  0.386E 00 -0.315E-02
  4 -0.177E 01 -0.692E-01 -0.315E-02
  5 -0.177E 01  0.821E-01 -0.315E-02

***************************************************************************

7. PILE FORCES ALONG PILE AXIS (KIPS & FT)

 PILE    F1       F3       F5               FAILURE
                                            BU CO TE
  1  -0.442  27.395   0.                    F
  2  -0.435  39.282   0.                    F
  3  -0.427  51.170   0.                    F
  4  -0.436  -9.167   0.
  5  -0.436  10.881   0.

TOTAL NO. FAILURES =  3       LOAD CASE 1

***************************************************************************

8. PILE FORCES ALONG STRUCTURE AXIS (KIPS & FEET)

 PILE      F1         F3         F5
  1     -9.082     25.849     0.
  2    -12.835     37.129     0.
  3    -16.587     48.408     0.
  4     -0.436     -9.167     0.
  5     -0.436     10.881     0.
---------------------------------------------------------------------------
SUM    -39.375    113.100    173.400

***************************************************************************
```

Example Problem 4

Two-dimensional problem, Hrennikoff's example case 4a (weak soil)

70. This example is also from Hrennikoff's paper, case 4a (weak soil). Figure 10 shows the physical problem. The properties and loading conditions are shown in Figure 12. The input data are presented in Table 8. The computer output is shown in Table 9.

71. This example serves as a means to verify that output agrees with the classical method.

Properties	
$E = 0.15 \times 10^7$ psi	Degree of fixity = 0.0
ES = 31.230 psi	Pile resistance (K2) = 1.0
$I_1 = 322.06$ in.^{4}	Participation factor for torsion (K4) = 0.0
$I_2 = 322.06$ in.^{4}	
Area = 63.5 in.^{2}	Torsion modulus = 0.0
Length = 30 ft	

Loading Case	Q_1 (kips)	Q_3 (kips)	Q_5 (kip-ft)
1	-39.375	113.1	173.4

Figure 12. Properties and loading conditions for example problem 4

Results and calculations

72. The pile forces along pile axis in the computer output presented in Table 9 agree closely with the results in Hrennikoff's (1950) paper, case 4a. For example, for pile 1 from the computer output

Table 8

Input Data for Example Problem 4

Group						
1A	10000	EXAMPLE PROBLEM NO. 4				
1B	10010	HRENNIKOFF'S EXAMPLE - CASE 4A				
2A	10020	2				
2B	10030	5	1	1		
3	10040	1	31.23			
4A	10050	1	5	30.000	3	
4D	10060	9.000				
5A	10070	4				
5C	10080	1500000.000				
6A	10090	2				
6C	10100	0.		1.000	0.	0.
7	10110	1				
8	10120	82.000		40.000	100.000	100.000
9A	10130	2				
9B	10140	1	3	-3.000		
	10150	4	5	0.		
10	10160	-5.000 -2.500 0. 3.000 7.000				
12	10170	-39.375		113.1	173.4	

Table 9

Output Data for Example Problem 4

```
EXAMPLE PROBLEM NO. 4
HRENNIKOFF'S EXAMPLE - CASE 4A

NO. OF PILE ROWS =    5    B MATRIX IS CALCULATED FOR EACH ROW


***********************************************************************


 1. TABLE OF PILE AND SOIL DATA

PILE NUMBERS

   1    5   E = 0.15E 07 PSI    IX =    322.06 IN**4    IY =    322.06 IN**4
            AREA =    63.6 IN**2    X =       9.00 IN      Y =     9.00 IN
            LENGTH =    30.0 FEET    ES =  31.230
            K1 =   0.4107  K2 =   1.0000  K3 =   0.
            K4 =   0.      K5 =   0.      K6 =   0.


         ALLOWABLES:  COMPRESSIVE LOAD =      82.000 KIPS
                      TENSILE LOAD =      40.000 KIPS
                      BENDING =     100.000 KIPS
                      MOMENT =     100.000 KIP-FT

 THE B MATRIX FOR PILES    1 THROUGH    5 IS

  0.138E 04  0.         0.
  0.         0.265E 06  0.
  0.         0.         0.


***********************************************************************


 2. TABLE OF PILE COORDINATES AND BATTER

PILE ROW  BATTER          U1 (FT)
    1      -3.00          -5.000
    2      -3.00          -2.500
    3      -3.00           0.
    4     VERTICAL         3.000
    5     VERTICAL         7.000


***********************************************************************


 3. STIFFNESS MATRIX S FOR THE STRUCTURE

  0.860E 05 -0.237E 06 -0.712E 07
 -0.237E 06  0.125E 07 -0.103E 08
 -0.712E 07 -0.103E 08  0.329E 10

 3A  FLEXIBILITY MATRIX F FOR THE STRUCTURE

  0.664E-04  0.142E-04  0.188E-06
  0.142E-04  0.386E-05  0.429E-07
  0.188E-06  0.429E-07  0.847E-09


COORDINATES OF ELASTIC CENTER
  EC1 =      0.003      EC2 =     -0.002
```

(Continued)

Table 9 (Concluded)

```
******** LOADING CONDITION 1 ********


*****************************************************************


4. MATRIX OF APPLIED LOADS Q (KIPS & FEET)

      Q1          Q3          Q5
   -39.375     113.100     173.400


*****************************************************************


5. STRUCTURE DEFLECTIONS (INCHES)

    D1          D3          D5
-0.616E 00 -0.332E-01 -0.805E-03


*****************************************************************


6. PILE DEFLECTIONS ALONG PILE AXIS (INCHES)

 PILE      X1          X3          X5
  1 -0.610E 00  0.117E 00 -0.805E-03
  2 -0.602E 00  0.140E 00 -0.805E-03
  3 -0.595E 00  0.163E 00 -0.805E-03
  4 -0.616E 00 -0.421E-02 -0.805E-03
  5 -0.616E 00  0.344E-01 -0.805E-03


*****************************************************************


7. PILE FORCES ALONG PILE AXIS (KIPS & FT)

 PILE   F1       F3       F5            FAILURE
                                        BU CO TE
  1  -0.845  31.132   0.                F
  2  -0.834  37.204   0.                F
  3  -0.824  43.276   0.                F
  4  -0.853  -1.117   0.
  5  -0.853   9.124   0.

TOTAL NO. FAILURES =  3      LOAD CASE 1


*****************************************************************


8. PILE FORCES ALONG STRUCTURE AXIS (KIPS & FEET)

 PILE     F1        F3        F5
  1    -10.646    29.267     0.
  2    -12.556    35.031     0.
  3    -14.467    40.795     0.
  4     -0.853    -1.117     0.
  5     -0.853     9.124     0.
-----------------------------------------------------------------
SUM    -39.375   113.100   173.400


*****************************************************************
```

$$F_1 = 0.845 \text{ kips} \quad \text{and} \quad F_3 = 31.13 \text{ kips} \quad F_5 = 0 \text{ kip-ft}$$

in comparison with

$$F_1 = 0.84 \text{ kips} \quad \text{and} \quad F_3 = 31.2 \text{ kips} \quad F_5 = 0 \text{ kip-ft}$$

from Hrennikoff's paper. The pile forces along the pile axis for piles 2-5 also agree closely. The computer results and Hrennikoff's results are presented below.

	Computer Output		Hrennikoff's Example	
Pile No.	F_1 (kips)	F_3 (kips)	F_1 (kips)	F_3 (kips)
1	0.845	31.132	0.84	31.2
2	0.834	37.204	0.83	37.2
3	0.824	43.276	0.82	43.2
4	0.853	-1.117	0.85	-1.0
5	0.853	9.124	0.85	9.1

Example Problem 5

Two-dimensional problem,
Hrennikoff's example
case 6a (medium soil)

73. This example is case 6a (medium soil) from Hrennikoff's paper. The physical problem is shown in Figure 10. The properties and loading conditions are shown in Figure 13. The input data are stored in a file prior to the run and are shown in Table 10. The computer output is presented in Table 11.

74. This example also serves as a means to verify that output agrees with the classical method results.

Properties	
$E = 0.15 \times 10^7$ psi	Degree of fixity = 0.0
ES = 312.30 psi	Pile resistance (K2) = 1.0
$I_1 = 322.06$ in.^{4}	Participation factor for torsion (K4) = 0.0
$I_2 = 322.06$ in.^{4}	
Area = 63.5 in.^{2}	Torsion modulus = 0.0
Length = 30 ft	

Loading Case	Q_1 (kips)	Q_3 (kips)	Q_5 (kip-ft)
1	-39.375	113.1	173.4

Figure 13. Properties and loading conditions for example problem 5

Results and calculations

75. Manual calculations for this example are presented in Hrennikoff's paper, case 6a. The computer results shown in Table 11 agree closely with the classical method results. For example, a comparison of the horizontal forces in each pile is shown below:

Table 10

Input Data for Example Problem 5

Group							
1A	10000	EXAMPLE PROBLEM NO. 5					
1B	10010	HRENNIKOFF'S EXAMPLE - CASE 6A					
2A	10020	2					
2B	10030	5	1	1			
3	10040	1	312.300				
4A	10050	1	5	30.000	3		
4D	10060	9.000					
5A	10070	4					
5C	10080	1500000.000					
6A	10090	2					
6C	10100	0.		1.000	0.	0.	
7	10110	1					
8	10120	82.000		40.000	100.000	100.000	
9A	10130	2					
9B	10140	1	3	-3.000			
	10150	4	5	0.			
10	10160	-5.000	-2.500	0.	3.000	7.000	
12	10170	-39.375		113.1	173.4		

Table 11

Output Data for Example Problem 5

```
EXAMPLE PROBLEM NO. 5
HRENNIKOFF'S EXAMPLE - CASE 6A

NO. OF PILE ROWS =   5     B MATRIX IS CALCULATED FOR EACH ROW


***********************************************************************


 1. TABLE OF PILE AND SOIL DATA

PILE NUMBERS

    1    5  E = 0.15E 07 PSI    IX =    322.06 IN**4    IY =   322.06 IN**4
            AREA =    63.6 IN**2    X =       9.00 IN      Y =      9.00 IN
            LENGTH =   30.0 FEET    ES = 312.300
            K1 =   0.4107  K2 =   1.0000  K3 =   0.
            K4 =   0.        K5 =   0.        K6 =   0.


         ALLOWABLES:  COMPRESSIVE LOAD =      82.000 KIPS
                      TENSILE LOAD =     40.000 KIPS
                      BENDING =     100.000 KIPS
                      MOMENT =     100.000 KIP-FT

 THE B MATRIX FOR PILES   1 THROUGH   5 IS

  0.779E 04  0.          0.
  0.         0.265E 06   0.
  0.         0.          0.


***********************************************************************


 2. TABLE OF PILE COORDINATES AND BATTER

PILE ROW  BATTER          U1 (FT)
    1      -3.00         -5.000
    2      -3.00         -2.500
    3      -3.00          0.
    4     VERTICAL        3.000
    5     VERTICAL        7.000


***********************************************************************


 3. STIFFNESS MATRIX S FOR THE STRUCTURE

  0.116E 06 -0.232E 06 -0.695E 07
 -0.232E 06  0.125E 07 -0.103E 08
 -0.695E 07 -0.103E 08  0.329E 10

 3A  FLEXIBILITY MATRIX F FOR THE STRUCTURE

  0.205E-04  0.427E-05  0.566E-07
  0.427E-05  0.171E-05  0.144E-07
  0.566E-07  0.144E-07  0.468E-09


COORDINATES OF ELASTIC CENTER
  EC1 =      0.003      EC2 =     -0.002
```

(Continued)

Table 11 (Concluded)

```
******** LOADING CONDITION 1 ********


**********************************************************************


4. MATRIX OF APPLIED LOADS Q (KIPS & FEET)

       Q1          Q3          Q5
    -39.375     113.100     173.400


**********************************************************************


5. STRUCTURE DEFLECTIONS (INCHES)

     D1          D3          D5
-0.207E 00  0.553E-01  0.368E-03


**********************************************************************


6. PILE DEFLECTIONS ALONG PILE AXIS (INCHES)

 PILE      X1          X3          X5
  1 -0.172E 00  0.139E 00  0.368E-03
  2 -0.175E 00  0.129E 00  0.368E-03
  3 -0.179E 00  0.118E 00  0.368E-03
  4 -0.207E 00  0.420E-01  0.368E-03
  5 -0.207E 00  0.243E-01  0.368E-03


**********************************************************************


7. PILE FORCES ALONG PILE AXIS (KIPS & FT)

 PILE    F1        F3        F5          FAILURE
                                         BU CO TE
  1   -1.338  36.790    0.
  2   -1.365  34.014    0.
  3   -1.392  31.237    0.
  4   -1.611  11.137    0.
  5   -1.611   6.454    0.

TOTAL NO. FAILURES =  0       LOAD CASE 1


**********************************************************************


8. PILE FORCES ALONG STRUCTURE AXIS (KIPS & FEET)

 PILE     F1          F3         F5
  1    -12.907     34.479      0.
  2    -12.051     31.836      0.
  3    -11.199     29.194      0.
  4     -1.611     11.137      0.
  5     -1.611      6.454      0.
----------------------------------------------------------------------
SUM    -39.375    113.100    173.400


**********************************************************************
```

Pile No.	F_1 (kips) from Computer Output	Hrennikoff's Example
1	1.338	1.34
2	1.365	1.37
3	1.392	1.39
4	1.611	1.61
5	1.611	1.61

76. The vertical pile forces also agree closely and are shown below:

Pile No.	F_3 (kips) from Computer Output	Hrennikoff's Example
1	36.790	36.8
2	34.014	34.0
3	31.237	31.2
4	11.137	11.1
5	6.454	6.5

Example Problem 6

Two-dimensional problem, 16 piles with linearly varying soil moduli

77. To further illustrate the use of program LMVDPILE for two-dimensional systems, a sixth example problem was run. The B-matrix terms are input directly. The physical problem is shown in Figure 14. The properties and loading conditions are shown in Figure 15. The input data are stored in a file that is listed in Table 12. The computer output is presented in Table 13.

78. This two-dimensional example was run to verify that the computer results agree with the St. Louis District's program output.

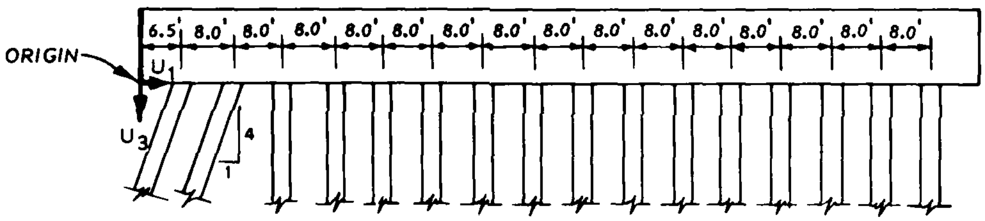

Figure 14. Physical problem for example problem 6

Properties	
$b_{11} = 5.6$	E = 30,000.0 psi
$b_{22} = 448.0$	KS = 0.001 pci
$b_{33} = 0.001$	
$b_{31} = 0$	

Loading Case	Q_1 (kips)	Q_3 (kips)	Q_5 (kip-ft)
1	-0.11	2.678	-177.5
2	-0.11	3.110	-206.67
3	-0.11	0.028	-1.233

Figure 15. Properties and loading conditions for example problem 6

Table 12

Input Data for Example Problem 6

Group								
1A	12000	EXAMPLE PROBLEM NO. 6						
1B	12010	JOHN H OVERTON LOCK AND DAM UFRAME ADJACENT						
2A	12020	2						
2B	12030	15 1 3						
3	12040	2 0.001						
4A	12050	1 10	60.000	1				
4B	12060	5.620	443.020	0.001	0.			
5A	12070	4						
5C	12080	30000.000						
6A	12090	1						
6B	12100	1.000	1.000	1.000	1.000	1.000	1.000	1.000
7	12110	1						
8	12120	0.365	0.250	0.1	0.1			
9A	12130	2						
9B	12140	1 2	-4.000					
	12150	3 16	0.					
10	12160	6.5,14.5,22.5,30.5,38.5,46.5,54.5,62.5,70.5,78.5,86.5,94.5,						
	12170	102.5,110.5,118.5,126.5						
12	12180	-0.110	2.673	-177.500				
	12190	-0.110	3.110	-226.667				
	12200	-0.110	0.025	-1.233				

Table 13

Output Data for Example Problem 6

```
EXAMPLE PROBLEM NO. 6
JOHN H OVERTON LOCK AND DAM UFRAME ADJACENT

NO. OF PILE ROWS =  16     B MATRIX IS CALCULATED FOR EACH ROW

*********************************************************************

1. TABLE OF PILE AND SOIL DATA

PILE NUMBERS

   1  16E = 0.30E 05 PSI    ES =     0.201
        K1 =1.2000  K2 =1.2000  K3 =1.2220
        K4 =1.0000  K5 =1.0000  K6 =1.2000

        ALLOWABLES:  COMPRESSIVE LOAD =      2.365 KIPS
                     TENSILE LOAD =      0.252 KIPS
                     BENDING =      0.122 KIPS
                     MOMENT =      0.120 KIP-FT

B MATRIX INPUT DIRECTLY AND THEN MODIFIED BY FIXITIES

THE B MATRIX FOR PILES   1 THROUGH  16 IS

 2.562E 01  0.          0.
 0.         2.446E 03   0.
 0.         0.          2.100E-02

*********************************************************************

2. TABLE OF PILE COORDINATES AND BATTER

PILE NO.  BATTER          X1 (FT)
   1       -4.00           6.502
   2       -4.20          14.520
   3      VERTICAL        22.502
   4      VERTICAL        30.520
   5      VERTICAL        3[illegible].520
   6      VERTICAL        46.500
   7      VERTICAL        54.522
   [illegible]      VERTICAL        6[illegible].520
   [illegible]      VERTICAL        70.522
  10      VERTICAL        7[illegible].520
  11      VERTICAL        [illegible].500
  12      VERTICAL        94.520
  13      VERTICAL       102.520
  14      VERTICAL       110.520
  15      VERTICAL       11[illegible].520
  16      VERTICAL       126.520

*********************************************************************

3. STIFFNESS MATRIX S FOR THE STRUCTURE

 2.142E 23 -2.229E 03  2.2[illegible]2F 05
-2.229E 23  0.712E 04 -2.[illegible]71E 07
 2.262E 05 -2.571E 07  0.597E 17

3A  FLEXIBILITY MATRIX F FOR THE STRUCTURE

 2.[illegible]25E-22  [illegible].[illegible]13E-03  2.[illegible]43E-06
 2.[illegible]14E-23  0.716E-03  2.[illegible]76E-06
 2.343E-06  2.676E-06  2.[illegible]11E-09

COORDINATES OF ELASTIC CENTER
  XC1 =      2.201      XC[illegible] =      0.002
```

(Continued)

(Sheet 1 of 7)

Table 13 (Continued)

```
******** LOADING CONDITION 1 ********


*********************************************************************


4. MATRIX OF APPLIED LOADS Q (KIPS & FEET)

       Q1           Q3          Q5
     -0.110        2.679     -177.520


*********************************************************************


5. STRUCTURE DEFLECTIONS (INCHES)

     D1           D3           D5
 -0.244E 00   0.361E 00 -0.103E-04


*********************************************************************


6. PILE DEFLECTIONS ALONG PILE AXIS (INCHES)

 PILE      X1            X3           X5
   1 -0.149E 00   0.410E 00 -0.103E-04
   2 -0.149E 00   0.411E 00 -0.103E-04
   3 -0.244E 00   0.364E 00 -0.123E-04
   4 -0.244E 00   0.365E 00 -0.103E-04
   5 -0.244E 00   0.366E 00 -0.103E-04
   6 -0.244E 00   0.367E 00 -0.103E-04
   7 -0.244E 00   0.368E 00 -0.103E-04
   8 -0.244E 00   0.369E 00 -0.103E-04
   9 -0.244E 00   0.370E 00 -0.123E-04
  10 -0.244E 00   0.371E 00 -0.123E-04
  11 -0.244E 00   0.372E 00 -0.103E-04
  12 -0.244E 00   0.373E 00 -0.103E-04
  13 -0.244E 00   0.374E 00 -0.123E-04
  14 -0.244E 00   0.375E 00 -0.123E-04
  15 -0.244E 00   0.376E 00 -0.103E-04
  16 -0.244E 00   0.377E 00 -0.123E-04


*********************************************************************
```

(Continued)

(Sheet 2 of 7)

Table 13 (Continued)

7. PILE FORCES ALONG PILE AXIS (KIPS & FT)

PILE	F1	F3	F5	FAILURE CODE
1	-0.001	0.154	-0.000	F
2	-0.001	0.154	-0.000	F
3	-0.001	0.163	-0.000	F
4	-0.001	0.163	-0.000	F
5	-0.001	0.154	-0.000	F
6	-0.001	0.164	-0.000	F
7	-0.001	0.165	-0.000	F
8	-0.001	0.165	-0.000	F
9	-0.001	0.166	-0.000	F
10	-0.001	0.166	-0.000	F
11	-0.001	0.166	-0.000	F
12	-0.001	0.167	-0.000	F
13	-0.001	0.167	-0.000	F
14	-0.001	0.168	-0.000	F
15	-0.001	0.168	-0.000	F
16	-0.001	0.169	-0.000	F

TOTAL NO. FAILURES = 16 LOAD CASE 1

****

8. PILE FORCES ALONG STRUCTURE AXIS (KIPS & FEET)

PILE	F1	F3	F5
1	-0.045	0.178	-0.000
2	-0.045	0.178	-0.000
3	-0.001	0.163	-0.000
4	-0.001	0.163	-0.000
5	-0.001	0.154	-0.000
6	-0.001	0.164	-0.000
7	-0.001	0.165	-0.000
8	-0.001	0.165	-0.000
9	-0.001	0.166	-0.000
10	-0.001	0.166	-0.000
11	-0.001	0.166	-0.000
12	-0.001	0.167	-0.000
13	-0.001	0.167	-0.000
14	-0.001	0.168	-0.000
15	-0.001	0.168	-0.000
16	-0.001	0.169	-0.000
SUM	-0.110	2.678	-177.500

****

(Continued)

(Sheet 3 of 7)

Table 13 (Continued)

******** LOADING CONDITION 2 ********

****

4. MATRIX OF APPLIED LOADS Q (KIPS & FEET)

Q1	Q3	Q5
-0.112	0.110	-225.667

****

5. STRUCTURE DEFLECTIONS (INCHES)

D1	D3	D5
-0.143E 00	0.431E 00	-0.219E-05

****

6. PILE DEFLECTIONS ALONG PILE AXIS (INCHES)

PILE	X1	X3	X5
1	-0.337E-01	0.453E 00	-0.219E-05
2	-0.336E-01	0.453E 00	-0.219E-05
3	-0.143E 00	0.432E 00	-0.219E-05
4	-0.143E 00	0.432E 00	-0.219E-05
5	-0.143E 00	0.432E 00	-0.219E-05
6	-0.143E 00	0.432E 00	-0.219E-05
7	-0.143E 00	0.433E 00	-0.219E-05
8	-0.143E 00	0.433E 00	-0.219E-05
9	-0.143E 00	0.433E 00	-0.219E-05
10	-0.143E 00	0.433E 00	-0.219E-05
11	-0.143E 00	0.433E 00	-0.219E-05
12	-0.143E 00	0.434E 00	-0.219E-05
13	-0.143E 00	0.434E 00	-0.219E-05
14	-0.143E 00	0.434E 00	-0.219E-05
15	-0.143E 00	0.434E 00	-0.219E-05
16	-0.143E 00	0.434E 00	-0.219E-05

****

(Continued)

(Sheet 4 of 7)

Table 13 (Continued)

7. PILE FORCES ALONG PILE AXIS (KIPS & FT)

PILE	F1	F3	F5	FAILURE CODE
1	-0.000	0.323	-0.000	F
2	-0.000	0.260	-0.000	F
3	-0.201	0.193	-0.000	F
4	-0.201	0.193	-0.000	F
5	-0.201	0.194	-0.000	F
6	-0.201	0.194	-0.000	F
7	-0.201	0.194	-0.000	F
8	-0.201	0.194	-0.000	F
9	-0.201	0.194	-0.000	F
10	-0.201	0.194	-0.000	F
11	-0.201	0.194	-0.000	F
12	-0.201	0.194	-0.000	F
13	-0.201	0.194	-0.000	F
14	-0.201	0.194	-0.000	F
15	-0.201	0.195	-0.000	F
16	-0.201	0.195	-0.000	F

TOTAL NO. FAILURES = 16   LOAD CASE 2

****

8. PILE FORCES ALONG STRUCTURE AXIS (KIPS & FEET)

PILE	F1	F3	F5
1	-0.249	0.197	-0.000
2	-0.249	0.197	-0.000
3	-0.201	0.193	-0.000
4	-0.201	0.193	-0.000
5	-0.201	0.194	-0.000
6	-0.201	0.194	-0.000
7	-0.201	0.194	-0.000
8	-0.201	0.194	-0.000
9	-0.201	0.194	-0.000
10	-0.201	0.194	-0.000
11	-0.201	0.194	-0.000
12	-0.201	0.194	-0.000
13	-0.201	0.194	-0.000
14	-0.201	0.194	-0.000
15	-0.201	0.195	-0.000
16	-0.201	0.195	-0.000
SUM	-0.110	3.110	-206.667

****

(Continued)

(Sheet 5 of 7)

Table 13 (Continued)

```
******** LOADING CONDITION 3 ********

*****************************************************************************

4. MATRIX OF APPLIED LOADS Q (KIPS & FEET)

       Q1           Q3          Q5
     -0.110        2.228       -1.233

*****************************************************************************

5. STRUCTURE DEFLECTIONS (INCHES)

     D1          D3          D5
 -0.895E 00 -0.911E-01 -0.858E-04

*****************************************************************************

6. PILE DEFLECTIONS ALONG PILE AXIS (INCHES)

 PILE      X1          X3          X5
   1 -0.888E 00  0.135E 00 -0.858E-04
   2 -0.866E 00  0.143E 00 -0.858E-04
   3 -0.895E 00 -0.680E-01 -0.858E-04
   4 -0.895E 00 -0.597E-01 -0.858E-04
   5 -0.895E 00 -0.515E-01 -0.858E-04
   6 -0.895E 00 -0.432E-01 -0.858E-04
   7 -0.895E 00 -0.350E-01 -0.858E-04
   8 -0.895E 00 -0.265E-01 -0.858E-04
   9 -0.895E 00 -0.185E-01 -0.858E-04
  10 -0.895E 00 -0.103E-01 -0.858E-04
  11 -0.895E 00 -0.207E-02 -0.858E-04
  12 -0.895E 00  0.517E-02 -0.858E-04
  13 -0.895E 00  0.144E-01 -0.858E-04
  14 -0.895E 00  0.226E-01 -0.858E-04
  15 -0.895E 00  0.309E-01 -0.858E-04
  16 -0.895E 00  0.391E-01 -0.858E-04

*****************************************************************************
```

(Continued)

Table 13 (Concluded)

7. PILE FORCES ALONG PILE AXIS (KIPS & FT)

PILE	F1	F3	F5	FAILURE BU CC TL
1	-0.005	0.061	-0.000	
2	-0.005	0.064	-0.000	
3	-0.005	-0.030	-0.000	
4	-0.005	-0.027	-0.000	
5	-0.005	-0.023	-0.000	
6	-0.005	-0.019	-0.000	
7	-0.005	-0.015	-0.000	
8	-0.005	-0.012	-0.000	
9	-0.005	-0.008	-0.000	
10	-0.005	-0.005	-0.000	
11	-0.005	-0.001	-0.000	
12	-0.005	0.003	-0.000	
13	-0.005	0.006	-0.000	
14	-0.005	0.010	-0.000	
15	-0.005	0.014	-0.000	
16	-0.005	0.018	-0.000	

TOTAL NO. FAILURES = 0 LOAD CASE 3

****

8. PILE FORCES ALONG STRUCTURE AXIS (KIPS & FEET)

PILE	F1	F3	F5
1	-0.022	0.057	-0.000
2	-0.022	0.061	-0.000
3	-0.005	-0.030	-0.000
4	-0.005	-0.027	-0.000
5	-0.005	-0.023	-0.000
6	-0.005	-0.019	-0.000
7	-0.005	-0.016	-0.000
8	-0.005	-0.012	-0.000
9	-0.005	-0.008	-0.000
10	-0.005	-0.005	-0.000
11	-0.005	-0.001	-0.000
12	-0.005	0.003	-0.000
13	-0.005	0.006	-0.000
14	-0.005	0.010	-0.000
15	-0.005	0.014	-0.000
16	-0.005	0.018	-0.000
SUM	-0.110	0.028	-1.233

****

ready

*

(Sheet 7 of 7)

Results and calculations

79. The computer results presented in Table 13 agree closely with answers from St. Louis program output. For example, for pile 1 and loading case 1, the pile forces along the structure axis from Table 13 are

$$F_1 = -0.045 \text{ kip}$$

$$F_3 = 0.178 \text{ kip}$$

$$F_5 = 0.0$$

as compared with

$$F_1 = -45.4 \text{ lb}$$

$$F_3 = 178.1 \text{ lb}$$

$$F_5 = 0$$

from the St. Louis program. The results for all piles agree very closely.

Example Problem 7

Three-dimensional
problem, 4 pinned piles
and constant soil modulus

80. This example problem illustrates the use of program LMVDPILE, given four vertical piles (similar to example problem 1 for 2-D system). Figures 16 and 17 show the physical problem. There are six loading conditions: a unit load applied along each axis, a unit moment about the U_1 and U_2 axes and a combination of all loads. Figure 18 shows the loading conditions and properties. The input data are stored in a data file prior to running the program and are shown in Table 14. The computer output is presented in Table 15.

81. This example illustrates how a three-dimensional problem with linearly varying soil modulus is coded. It also serves as a means to verify the computer output by comparison with manual calculations.

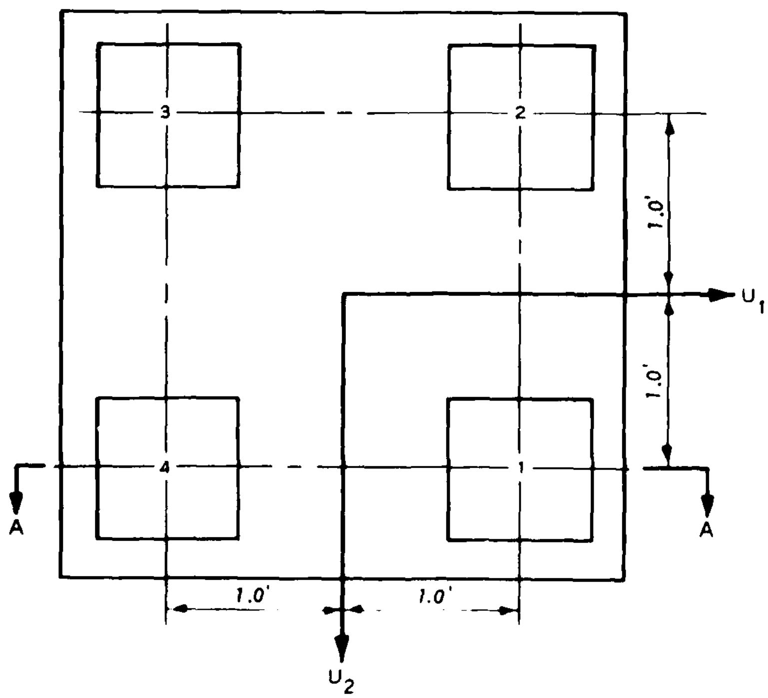

Figure 16. Plan view of example problem 7

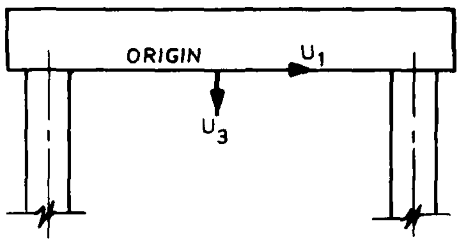

Figure 17. Section A-A for example problem 7

Properties	
Ult. str. of concrete = 5000 psi	Vertical (h = 0.0)
KS = 10.000 pci	
I_1 = 833.333 in.^{4}	Degree of fixity = 0.0
I_2 = 833.333 in.^{4}	Pile resistance (K2) = 1.0
Area = 100.0 in.^{2}	Participation factor for torsion (K4) = 0.0
Length = 100.0 ft	Torsion modulus = 0.0

Loading Case	Q_1 (kips)	Q_2 (kips)	Q_3 (kips)	Q_4 (kip-ft)	Q_5 (kip-ft)	Q_6 (kip-ft)
1	1.0	0.0	0.0	0.0	0.0	0.0
2	0.0	1.0	0.0	0.0	0.0	0.0
3	0.0	0.0	1.0	0.0	0.0	0.0
4	0.0	0.0	0.0	1.0	0.0	0.0
5	0.0	0.0	0.0	0.0	1.0	0.0
6	1.0	1.0	1.0	1.0	1.0	0.0

Figure 18. Properties and loading conditions for example problem 7

Table 14

Input Data for Example Problem 7

Group			
1A	10000	EXAMLE PROBLEM NO. 7	TITLE
1B	10010	VERTICAL PILES WITH UNIT LOADS	
2A	10020	3	TYPE OF ANALYSIS 3-D
2B	10030	4 1 6	NUMBER OF PILES, PILE GROUPS, LOADING CONDITIONS
3	10040	2 10.000	SOIL PROPERTIES
4A	10050	1 4 100.000 2	
4C	10060	833.333 833.333 100.000 10.000 10.000	PILE GEOMETRY
5A	10070	1	
5B	10080	5000.000 150.000	PILE MATERIAL
6A	10090	2	
6C	10100	0. 1.000 0. 0.	PILE FIXITIES
7	10110	100.00 100.00 100.00 100.00 100.00 100.00 100.00 100.00	ALLOWABLE LOAD & MOMENTS
8A	10120	1	
8B	10130	1 4 0. 0.	BATTER AND ANGLE ORIENTATION
9A	10140	2*1.000,2*-1.000	U1 COORDINATES
9B	10150	1.000 2*-1.000 1.000	U2 COORDINATES
9C	10160	4*0.0	U3 COORDINATES
	10180	1.000 0. 0. 0. 0. 0.	
	10190	0. 1.000 0. 0. 0. 0.	
11	10200	0. 0. 1.000 0. 0. 0.	APPLIED LOADS AND MOMENTS
	10210	0. 0. 0. 1.000 0. 0.	
	10220	0. 0. 0. 0. 1.000 0.	
	10230	1.000 1.000 1.000 1.000 1.000 0.	

Table 15

Output Data for Example Problem 7

```
EXAMLE PROBLEM NO. 7
VERTICAL PILES WITH UNIT LOADS

NO. OF PILES =   4     B MATRIX IS CALCULATED FOR EACH PILE

**************************************************************************

1. TABLE OF PILE AND SOIL DATA

PILE NUMBERS

   1    4  E = 0.43E 07 PSI    IX =    833.33 IN**4    IY =    833.33 IN**4
           AREA =  100.0 IN**2    X =     10.00 IN      Y =     10.00 IN
           LENGTH =  100.0 FEET    ES =  10.000
           K1 =    0.4107  K2 =    1.0000  K3 =    0.
           K4 =    0.      K5 =    0.      K6 =    0.


ALLOWABLES:  COMBINED BENDING FOR TENSION =    100.000 KIPS
             MOMENT ABOUT MINOR AXIS FOR TENSION =    100.000 KIP-FT
             MOMENT ABOUT MAJOR AXIS FOR TENSION =    100.000 KIP-FT
             COMBINED BENDING FOR COMPRESSION =    100.000 KIPS
             MOMENT ABOUT MINOR AXIS FOR COMPRESSION =    100.000 KIP-FT
             MOMENT ABOUT MAJOR AXIS FOR COMPRESSION =    100.000 KIP-FT
             COMPRESSIVE LOAD =    100.000 KIPS
             TENSILE LOAD =    100.000 KIPS

THE B MATRIX FOR PILES    1 THROUGH    4 IS

 0.108E 05  0.         0.         0.         0.         0.
 0.         0.108E 05  0.         0.         0.         0.
 0.         0.         0.357E 06  0.         0.         0.
 0.         0.         0.         0.         0.         0.
 0.         0.         0.         0.         0.         0.
 0.         0.         0.         0.         0.         0.


**************************************************************************

2. TABLE OF PILE COORDINATES AND BATTER

PILE NO.  BATTER    ANGLE  U1(FT)  U2(FT)  U3(FT)
   1      VERTICAL   0.    1.000   1.000   0.
   2      VERTICAL   0.    1.000  -1.000   0.
   3      VERTICAL   0.   -1.000  -1.000   0.
   4      VERTICAL   0.   -1.000   1.000   0.


**************************************************************************

3. STIFFNESS MATRIX S FOR THE STRUCTURE

 0.433E 05  0.         0.         0.         0.         0.
 0.         0.433E 05  0.         0.         0.         0.
 0.         0.         0.143E 07  0.         0.         0.
 0.         0.         0.         0.206E 09  0.         0.
 0.         0.         0.         0.         0.206E 09  0.
 0.         0.         0.         0.         0.         0.125E 08

3A  FLEXIBILITY MATRIX F FOR THE STRUCTURE

 0.231E-04  0.         0.         0.         0.         0.
 0.         0.231E-04  0.         0.         0.         0.
 0.         0.         0.700E-06  0.         0.         0.
 0.         0.         0.         0.486E-08  0.         0.
 0.         0.         0.         0.         0.486E-08  0.
 0.         0.         0.         0.         0.         0.801E-07
```

(Continued)

(Sheet 1 of 7)

Table 15 (Continued)

******** LOADING CONDITION 1 ********

****

4. MATRIX OF APPLIED LOADS Q (KIPS & FEET)

Q1	Q2	Q3	Q4	Q5	Q6
1.000	0.	0.	0.	0.	0.

****

5. STRUCTURE DEFLECTIONS (INCHES)

D1	D2	D3	D4	D5	D6
0.231E-01	0.	0.	0.	0.	0.

****

6. PILE DEFLECTIONS ALONG PILE AXIS (INCHES)

PILE	X1	X2	X3	X4	X5	X6
1	0.231E-01	0.	0.	0.	0.	0.
2	0.231E-01	0.	0.	0.	0.	0.
3	0.231E-01	0.	0.	0.	0.	0.
4	0.231E-01	0.	0.	0.	0.	0.

****

7. PILE FORCES ALONG PILE AXIS (KIPS & FEET)

PILE	F1	F2	F3	F4	F5	F6	CBFTR	FAILURE CB BJ CO TE
1	0.250	0.	0.	0.	0.	0.	0.	
2	0.250	0.	0.	0.	0.	0.	0.	
3	0.250	0.	0.	0.	0.	0.	0.	
4	0.250	0.	0.	0.	0.	0.	0.	

TOTAL NO. FAILURES = 0 LOAD CASE 1

****

8. PILE FORCES ALONG STRUCTURE AXIS (KIPS & FEET)

PILE	F1	F2	F3	F4	F5	F6
1	0.250	0.	0.	0.	0.	0.
2	0.250	0.	0.	0.	0.	0.
3	0.250	0.	0.	0.	0.	0.
4	0.250	0.	0.	0.	0.	0.
SUM	1.000	0.	0.	0.	0.	0.000

****

(Continued)

(Sheet 2 of 7)

Table 15 (Continued)

```
****** LOADING CONDITION 2 ********


*********************************************************************

4. MATRIX OF APPLIED LOADS Q (KIPS & FEET)

       Q1            Q2           Q3            Q4            Q5            Q6
      0.            1.000        0.            0.            0.            0.


*********************************************************************

5. STRUCTURE DEFLECTIONS (INCHES)

    D1            D2           D3            D4            D5            D6
 0.           0.231E-01   0.            0.            0.            0.


*********************************************************************

6. PILE DEFLECTIONS ALONG PILE AXIS (INCHES)

 PILE      X1           X2           X3           X4           X5           X6
  1  0.          0.231E-01  0.           0.           0.           0.
  2  0.          0.231E-01  0.           0.           0.           0.
  3  0.          0.231E-01  0.           0.           0.           0.
  4  0.          0.231E-01  0.           0.           0.           0.


*********************************************************************

7. PILE FORCES ALONG PILE AXIS (KIPS & FEET)

 PILE    F1       F2       F3       F4       F5       F6     CBFTR    FAILURE
                                                                     CB BJ CO TE
  1   0.      0.250   0.       0.       0.       0.       0.
  2   0.      0.250   0.       0.       0.       0.       0.
  3   0.      0.250   0.       0.       0.       0.       0.
  4   0.      0.250   0.       0.       0.       0.       0.
 TOTAL NO. FAILURES =  0       LOAD CASE 2


*********************************************************************

8. PILE FORCES ALONG STRUCTURE AXIS (KIPS & FEET)

 PILE    F1          F2          F3          F4          F5          F6
  1    0.         0.250      0.          0.          0.          0.
  2    0.         0.250      0.          0.          0.          0.
  3    0.         0.250      0.          0.          0.          0.
  4    0.         0.250      0.          0.          0.          0.
 ---------------------------------------------------------------------
 SUM   0.         1.000      0.          0.          0.          0.000


*********************************************************************
```

(Continued)

(Sheet 3 of 7)

Table 15 (Continued)

```
******** LOADING CONDITION 3 ********

************************************************************************

4. MATRIX OF APPLIED LOADS Q (KIPS & FEET)
       Q1           Q2           Q3          Q4          Q5          Q6
      0.           0.          1.000         0.          0.          0.

************************************************************************

5. STRUCTURE DEFLECTIONS (INCHES)
    D1          D2          D3          D4          D5          D6
  0.          0.       0.700E-03  0.          0.          0.

************************************************************************

6. PILE DEFLECTIONS ALONG PILE AXIS (INCHES)
  PILE      X1          X2          X3          X4          X5          X6
   1  0.          0.       0.700E-03  0.          0.          0.
   2  0.          0.       0.700E-03  0.          0.          0.
   3  0.          0.       0.700E-03  0.          0.          0.
   4  0.          0.       0.700E-03  0.          0.          0.

************************************************************************

7. PILE FORCES ALONG PILE AXIS (KIPS & FEET)
  PILE   F1       F2       F3       F4       F5       F6     CBFTR   FAILURE
                                                                   CB BU CO TF
   1   0.       0.      0.250    0.       0.       0.       0.00
   2   0.       0.      0.250    0.       0.       0.       0.00
   3   0.       0.      0.250    0.       0.       0.       0.00
   4   0.       0.      0.250    0.       0.       0.       0.00
  TOTAL NO. FAILURES =  0      LOAD CASE 3

************************************************************************

8. PILE FORCES ALONG STRUCTURE AXIS (KIPS & FEET)
  PILE     F1         F2         F3         F4         F5         F6
   1      0.         0.        0.250      0.         0.         0.
   2      0.         0.        0.250      0.         0.         0.
   3      0.         0.        0.250      0.         0.         0.
   4      0.         0.        0.250      0.         0.         0.
  ---------------------------------------------------------------------
  SUM     0.         0.        1.000     -0.000     -0.000      0.

************************************************************************
```

(Continued)

(Sheet 4 of 7)

Table 15 (Continued)

******** LOADING CONDITION 4 ********

****

4. MATRIX OF APPLIED LOADS Q (KIPS & FEET)

Q1	Q2	Q3	Q4	Q5	Q6
0.	0.	0.	1.000	0.	0.

****

5. STRUCTURE DEFLECTIONS (INCHES)

D1	D2	D3	D4	D5	D6
0.	0.	0.	0.583E-04	0.	0.

****

6. PILE DEFLECTIONS ALONG PILE AXIS (INCHES)

PILE	X1	X2	X3	X4	X5	X6
1	0.	0.	0.700E-03	0.583E-04	0.	0.
2	0.	0.	-0.700E-03	0.583E-04	0.	0.
3	0.	0.	-0.700E-03	0.583E-04	0.	0.
4	0.	0.	0.700E-03	0.583E-04	0.	0.

****

7. PILE FORCES ALONG PILE AXIS (KIPS & FEET)

PILE	F1	F2	F3	F4	F5	F6	CBFTR	FAILURE CP BU CO TF
1	0.	0.	0.250	0.	0.	0.	0.00	
2	0.	0.	-0.250	0.	0.	0.	0.00	
3	0.	0.	-0.250	0.	0.	0.	0.00	
4	0.	0.	0.250	0.	0.	0.	0.00	

TOTAL NO. FAILURES = 0 LOAD CASE 4

****

8. PILE FORCES ALONG STRUCTURE AXIS (KIPS & FEET)

PILE	F1	F2	F3	F4	F5	F6
1	0.	0.	0.250	0.	0.	0.
2	0.	0.	-0.250	0.	0.	0.
3	0.	0.	-0.250	0.	0.	0.
4	0.	0.	0.250	0.	0.	0.
SUM	0.	0.	-0.000	1.000	-0.000	0.

****

(Continued)

(Sheet 5 of 7)

Table 15 (Continued)

******** LOADING CONDITION 5 ********

****

4. MATRIX OF APPLIED LOADS Q (KIPS & FEET)

Q1	Q2	Q3	Q4	Q5	Q6
0.	0.	0.	0.	1.000	0.

****

5. STRUCTURE DEFLECTIONS (INCHES)

D1	D2	D3	D4	D5	D6
0.	0.	0.	0.	0.583E-04	0.

****

6. PILE DEFLECTIONS ALONG PILE AXIS (INCHES)

PILE	X1	X2	X3	X4	X5	X6
1	0.	0.	-0.700E-03	0.	0.583E-04	0.
2	0.	0.	-0.700E-03	0.	0.583E-04	0.
3	0.	0.	0.700E-03	0.	0.583E-04	0.
4	0.	0.	0.700E-03	0.	0.583E-04	0.

****

7. PILE FORCES ALONG PILE AXIS (KIPS & FEET)

PILE	F1	F2	F3	F4	F5	F6	CBFTR	FAILURE
		0						CB BU CO TE
1	0.	0.	-0.250	0.	0.	0.	0.00	
2	0.	0.	-0.250	0.	0.	0.	0.00	
3	0.	0.	0.250	0.	0.	0.	0.00	
4	0.	0.	0.250	0.	0.	0.	0.00	

TOTAL NO. FAILURES = 0 LOAD CASE 5

****

8. PILE FORCES ALONG STRUCTURE AXIS (KIPS & FEET)

PILE	F1	F2	F3	F4	F5	F6
1	0.	0.	-0.250	0.	0.	0.
2	0.	0.	-0.250	0.	0.	0.
3	0.	0.	0.250	0.	0.	0.
4	0.	0.	0.250	0.	0.	0.
SUM	0.	0.	-0.000	-0.000	1.000	0.

****

(Continued)

(Sheet [illegible])

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DOCUMENTATION FOR LMVDPILE PROGRAM.(U)
JUN 80 D K MARTIN, H W JONES, N RADHAKRISHNAN
UNCLASSIFIED WES-TR-K-80-3   NL

2 OF 3
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A087191

Table 15 (Concluded)

******** LOADING CONDITION 6 ********

****

4. MATRIX OF APPLIED LOADS Q (KIPS & FEET)

Q1	Q2	Q3	Q4	Q5	Q6
1.000	1.000	1.000	1.000	1.000	0.

****

5. STRUCTURE DEFLECTIONS (INCHES)

D1	D2	D3	D4	D5	D6
0.231E-01	0.231E-01	0.700E-03	0.583E-04	0.583E-04	0.

****

6. PILE DEFLECTIONS ALONG PILE AXIS (INCHES)

PILE	X1	X2	X3	X4	X5	X6
1	0.231E-01	0.231E-01	0.700E-03	0.583E-04	0.583E-04	0.
2	0.231E-01	0.231E-01	-0.700E-03	0.583E-04	0.583E-04	0.
3	0.231E-01	0.231E-01	0.700E-03	0.583E-04	0.583E-04	0.
4	0.231E-01	0.231E-01	0.210E-02	0.583E-04	0.583E-04	0.

****

7. PILE FORCES ALONG PILE AXIS (KIPS & FEET)

PILE	F1	F2	F3	F4	F5	F6	CBFTA	FAILURE CB BU CO TE
1	0.250	0.250	0.250	0.	0.	0.	0.00	
2	0.250	0.250	-0.250	0.	0.	0.	0.00	
3	0.250	0.250	0.250	0.	0.	0.	0.00	
4	0.250	0.250	0.750	0.	0.	0.	0.01	

TOTAL NO. FAILURES = 0 LOAD CASE 6

****

8. PILE FORCES ALONG STRUCTURE AXIS (KIPS & FEET)

PILE	F1	F2	F3	F4	F5	F6
1	0.250	0.250	0.250	0.	0.	0.
2	0.250	0.250	-0.250	0.	0.	0.
3	0.250	0.250	0.250	0.	0.	0.
4	0.250	0.250	0.750	0.	0.	0.
SUM	1.000	1.000	1.000	1.000	1.000	0.000

****

(Sheet 7 of 7)

Results and calculations

82. The pile forces can be calculated by satisfying equilibrium $\Sigma F = 0$. These were found to agree with the program output shown in Table 15. For example, in loading case 3 with a 1-kip vertical load, the force in each pile is 0.25 kip. The force on each pile is

$$F_H = 1/4 \text{ (applied vertical load)} = 1/4 \text{ (1 kip)} = 0.25 \text{ kip}$$

The displacement in each pile is equal to

$$\delta = \frac{PL}{AE} = \frac{\frac{1}{4} \times 1 \times 100 \times 12}{4300 \times 144 \times \frac{100}{144}} = 0.7 \times 10^{-3} \text{ in.}$$

This result also agrees with the computer program results.

83. In loading case 4, a 1 kip-ft moment is applied about the U_1-axis. The pile forces can be calculated by satisfying equilibrium $M_{U_1} = 0$.

$$\Sigma M_{U_1} = F3_1 U_1(1) - F3_2 U_1(2) - F3_3 U_1(2) + F3_4 U_1(1) + Q_4$$

where

$F3_n$ = vertical force in pile n, n = 1 - 4

Q_4 = applied moment = 1 kip-ft

U_1 = horizontal distance along the U_1-axis

$$U_1(1) = 1.0 \text{ ft}$$

$$U_1(2) = 1.0 \text{ ft}$$

$$\therefore \Sigma M_{U_1} = F3_1 - F3_2 - F3_3 + F3_4 + 1.0 = 0$$

From symmetry $F3_1 = F3_4$ and $F3_2 = F3_3$

$$\therefore |F3_n| = 0.25 \text{ kip}$$

This result agrees with the computer output.

84. Load case 6 can be obtained as a superposition of load cases 1 through 5. The deflections of the piles and the load on each pile can also be obtained by superimposing the respective results for load cases 1 through 5. The following computations verify the computer results in item 6 (deflections) and item 8 (loads).

Pile No.	Load Case	Deflections X_1 (in.)	X_2 (in.)	X_3 (in.)	X_4 (rad)	X_5 (rad)
1	1	0.0231	0.	0.	0.	0.
	2	0.	0.0231	0.	0.	0.
	3	0.	0.	0.7×10^{-3}	0.	0.
	4	0.	0.	0.7×10^{-3}	0.583×10^{-4}	0.
	5	0.	0.	-0.7×10^{-3}	0.	0.583×10^{-4}
	6	0.0231	0.0231	0.7×10^{-3}	0.583×10^{-4}	0.583×10^{-4}
2	1	0.0231	0.	0.	0.	0.
	2	0.	0.0231	0.	0.	0.
	3	0.	0.	0.7×10^{-3}	0.	0.
	4	0.	0.	-0.7×10^{-3}	0.583×10^{-4}	0.
	5	0.	0.	-0.7×10^{-3}	0.	0.583×10^{-4}
	6	0.0231	0.0231	-0.7×10^{-3}	0.583×10^{-4}	0.583×10^{-4}
3	1	0.0231	0.	0.	0.	0.
	2	0.	0.0231	0.	0.	0.
	3	0.	0.	0.7×10^{-3}	0.	0.
	4	0.	0.	-0.7×10^{-3}	0.583×10^{-4}	0.
	5	0.	0.	0.7×10^{-3}	0.	0.583×10^{-4}
	6	0.0231	0.0231	0.7×10^{-3}	0.583×10^{-4}	0.583×10^{-4}
4	1	0.0231	0.	0.	0.	0.
	2	0.	0.0231	0.	0.	0.
	3	0.	0.	0.7×10^{-3}	0.	0.
	4	0.	0.	0.7×10^{-3}	0.583×10^{-4}	0.
	5	0.	0.	0.7×10^{-3}	0.	0.583×10^{-4}
	6	0.0231	0.0231	0.21×10^{-2}	0.583×10^{-4}	0.583×10^{-4}

(Continued)

Pile No.	Load Case	Loads F_1 (kips)	F_2 (kips)	F_3 (kips)	F_4 (kip-ft)	F_5 (kip-ft)
1	1	0.25	0.	0.	0.	0.
	2	0.	0.25	0.	0.	0.
	3	0.	0.	0.25	0.	0.
	4	0.	0.	0.25	0.	0.
	5	0.	0.	-0.25	0.	0.
	6	0.25	0.25	0.25	0.	0.
2	1	0.25	0.	0.	0.	0.
	2	0.	0.25	0.	0.	0.
	3	0.	0.	0.25	0.	0.
	4	0.	0.	-0.25	0.	0.
	5	0.	0.	-0.25	0.	0.
	6	0.25	0.25	-0.25	0.	0.
3	1	0.25	0.	0.	0.	0.
	2	0.	0.25	0.	0.	0.
	3	0.	0.	0.25	0.	0.
	4	0.	0.	-0.25	0.	0.
	5	0.	0.	0.25	0.	0.
	6	0.25	0.25	0.25	0.	0.
4	1	0.25	0.	0.	0.	0.
	2	0.	0.25	0.	0.	0.
	3	0.	0.	0.25	0.	0.
	4	0.	0.	0.25	0.	0.
	5	0.	0.	0.25	0.	0.
	6	0.25	0.25	0.75	0.	0.

These results also agree with the computer program results.

Example Problem 8

Three-dimensional problem, 1 fixed vertical pile

85. This example problem has only one vertical pile completely fixed into the rigid cap. It is similar to example 2 except the analysis now is three-dimensional. Figure 19 shows the physical problem. A 1 kip-ft moment is applied about the U_1, U_2, and U_3 axes. Figure 20 shows the loading conditions and properties. The input data are stored in a file prior to running the program and are presented in Table 16. The computer output is presented in Table 17.

86. This example serves as a means to verify the computer output by comparison with manual calculations.

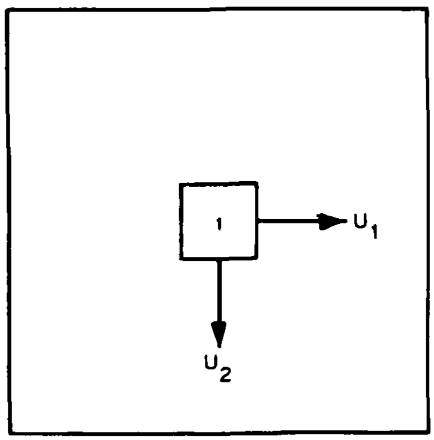

Figure 19. Plan view for example problem 8

Results and calculations

87. In this example, 1 kip-ft moments about the U_1, U_2, and U_3 axes were applied at the center of the structure where the pile is located. The pile is completely fixed into the rigid cap. Therefore, the resulting moments about the U_1, U_2, and U_3 axes are 1 kip-ft. These results agree with the program output presented in Table 17.

Properties	
Ult. str. of concrete = 5000 psi	K_1 = 1.0756
KS = 10.000 pci	K_2 = 1.0
I_1 = 833.333 in.^{4}	K_3 = 1.4988
I_2 = 833.333 in.^{4}	K_4 = 1.000
Area = 100.0 in.^{2}	K_5 = 0.9990
Length = 100.0 ft	K_6 = 0.9990
Vertical = (h = 0.0)	

Loading Case	Q_1 (kips)	Q_2 (kips)	Q_3 (kips)	Q_4 (kip-ft)	Q_5 (kip-ft)	Q_6 (kip-ft)
1	0.0	0.0	0.0	1.0	1.0	1.0

Figure 20. Properties and loading conditions for example problem 8

Table 16

Input Data for Example Problem 8

Group									
1A	10000	EXAMPLE PROBLEM NO. 8							
1B	10010	ONE FIXED VERTICAL PILE WITH UNIT MOMENTS APPLIED							
2A	10020	3							
2B	10030	1	1	1					
3	10040	2	10.000						
4A	10050	1	1	100.000	2				
4C	10060	833.333	833.333	100.000	10.000	10.000			
5A	10070	1							
5B	10080	5000.000	150.000						
6A	10090	1							
6B	10100	1.000	1.0756	1.000	1.4938	1.000	0.999	0.999	
7	10110	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
8A	10120	0							
10	10130	0.	0.	0.	0.	0.			
11	10140	0.	0.	0.	1.000	1.000	1.000		

read/

*

Table 17

Output Data for Example Problem 8

```
EXAMPLE PROBLEM NO. 8
ONE FIXED VERTICAL PILE WITH UNIT MOMENTS APPLIED

NO. OF PILES =   1     B MATRIX IS CALCULATED FOR EACH PILE

**********************************************************************

1. TABLE OF PILE AND SOIL DATA

PILE NUMBERS

   1   1  E = 0.432E 07 PSI    IX =    833.33 IN**4    IY =    833.33 IN**4
          AREA =  100.0 IN**2   X =      12.00 IN       Y =     12.00 IN
          LENGTH =  100.0 FEET    ES =  10.000
          K1 =   1.0756  K2 =   1.0000  K3 =   1.4998
          K4 =   1.0000  K5 =   0.9990  K6 =   0.9990

ALLOWABLES:  COMBINED BENDING FOR TENSION =   100.000 KIPS
             MOMENT ABOUT MINOR AXIS FOR TENSION =    100.000 KIP-FT
             MOMENT ABOUT MAJOR AXIS FOR TENSION =    100.000 KIP-FT
             COMBINED BENDING FOR COMPRESSION =   100.000 KIPS
             MOMENT ABOUT MINOR AXIS FOR COMPRESSION =    100.000 KIP-FT
             MOMENT ABOUT MAJOR AXIS FOR COMPRESSION =    100.000 KIP-FT
             COMPRESSIVE LOAD =   100.000 KIPS
             TENSILE LOAD =   100.000 KIPS

THE B MATRIX FOR PILES   1 THROUGH   1 IS

 0.234E 05  0.          0.          0.          0.135E 07  0.
 0.         0.234E 05   0.         -0.135E 07   0.          0.
 0.         0.          0.357E 06   0.          0.          0.
 0.        -0.135E 07   0.          0.104E 09   0.          0.
 0.135E 07  0.          0.          0.          0.104E 09   0.
 0.         0.          0.          0.          0.          0.100E 04

**********************************************************************

2. TABLE OF PILE COORDINATES AND BATTER

PILE NO.  BATTER   ANGLE  U1(FT)  U2(FT)  U3(FT)
   1      VERTICAL   0.    0.       0.      0.

**********************************************************************

3. STIFFNESS MATRIX S FOR THE STRUCTURE

 0.234E 05  0.          0.          0.          0.135E 07  0.
 0.         0.234E 05   0.         -0.135E 07   0.          0.
 0.         0.          0.357E 06   0.          0.          0.
 0.        -0.135E 07   0.          0.104E 09   0.          0.
 0.135E 07  0.          0.          0.          0.104E 09   0.
 0.         0.          0.          0.          0.          0.100E 04

3A  FLEXIBILITY MATRIX F FOR THE STRUCTURE

 0.925E-04  0.          0.          0.         -0.120E-05   0.
 0.         0.925E-04   0.          0.120E-05   0.          0.
 0.         0.          0.280E-05   0.          0.          0.
 0.         0.120E-05   0.          0.252E-07   0.          0.
-0.120E-05  0.          0.          0.          0.252E-07   0.
 0.         0.          0.          0.          0.          0.100E-02
```

(Continued)

Table 17 (Concluded)

```
******** LOADING CONDITION 1 ********


*********************************************************************


4. MATRIX OF APPLIED LOADS Q (KIPS & FEET)

        Q1            Q2            Q3            Q4            Q5            Q6
       0.            0.            0.            1.000         1.000         1.200


*********************************************************************


5. STRUCTURE DEFLECTIONS (INCHES)

    D1          D2          D3          D4          D5          D6
-0.144E-01  0.144E-01  0.          0.302E-03  0.322E-03  0.122E 02


*********************************************************************


6. PILE DEFLECTIONS ALONG PILE AXIS (INCHES)

 PILE     X1          X2          X3          X4          X5          X6
  1 -0.144E-01  0.144E-01  0.          2.302E-03  0.302E-03  0.120E 02


*********************************************************************


7. PILE FORCES ALONG PILE AXIS (KIPS & FEET)

 PILE   F1       F2       F3       F4       F5       F6    CBFTR   FAILURE
                                                                  CB BU CO TE
  1    0.000   -0.000   0.       1.000    1.000    1.000   0.22

TOTAL NO. FAILURES =  0      LOAD CASE 1


*********************************************************************


8. PILE FORCES ALONG STRUCTURE AXIS (KIPS & FEET)

 PILE     F1       F2       F3       F4       F5       F6
  1      0.000   -0.000    0.       1.000    1.000    1.000
---------------------------------------------------------------------
 SUM     0.000   -0.000    0.       1.200    1.000    1.070


*********************************************************************
```

Example Problem 9

Three-dimensional
problem, 27 piles
with constant soil modulus

88. To demonstrate further the use of program LMVDPILE an example problem with a constant soil modulus is given. Figure 21 shows the physical problem for this example. The properties and loading conditions are presented in Figure 22. The input data are input interactively and are shown in Table 18. These data are saved in a file and are listed in Table 19. The computer output is presented in Table 20.

89. This example illustrates the option of inputting data interactively for a three-dimensional problem with 27 piles (vertical and battered) and a constant soil modulus. It also shows how the batter can be input in groups.

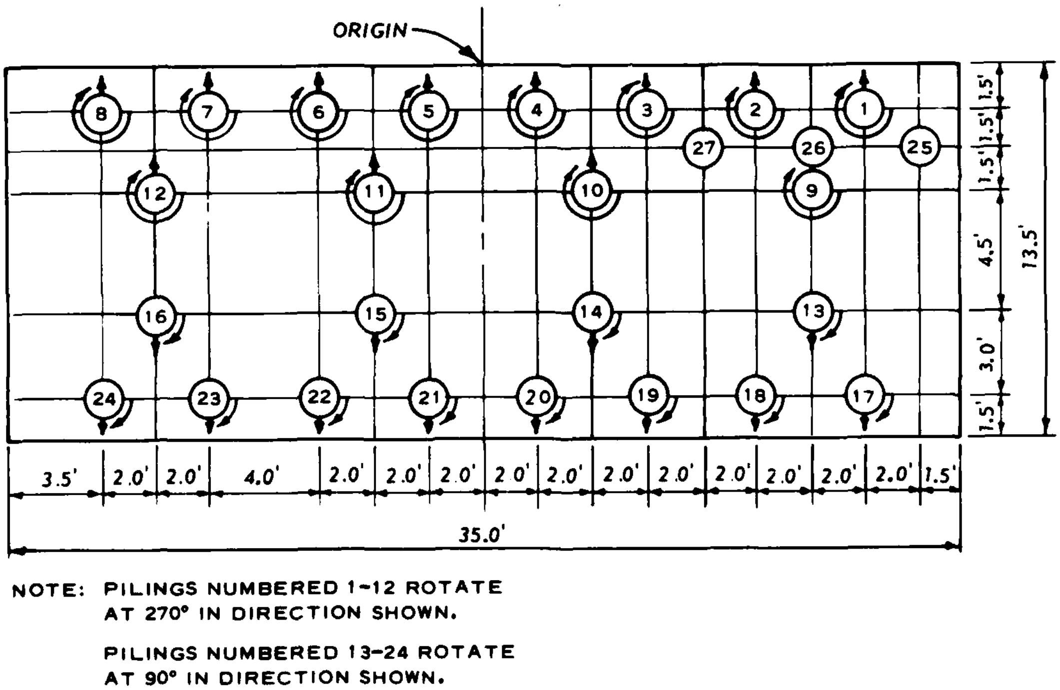

Figure 21. Physical problem for example problem 9

Properties	
Ult. str. of concrete = 5000 psi	K_1 = 0.4107
ES = 200.0 psi	K_2 = 1.0
I_1 = 1728.0 in.^{4} DF = 0.0	K_3-K_6 = 0.0
I_2 = 1728.0 in.^{4} PR = 1.0	
Area = 144.0 in.^{2} PFT = 0.0	
Length = 70.0 ft G = 0.0	

Loading Case	Q_1 (kips)	Q_2 (kips)	Q_3 (kips)	Q_4 (kip-ft)	Q_5 (kip-ft)	Q_6 (kip-ft)
1	0.0	276.961	344.9	5287.422	0.0	0.0

Figure 22. Properties and loading conditions for example problem 9

Results and calculations

90. The program output is shown in Table 20. From statics, $\Sigma F = 0$.

$$\Sigma F_1 = Q1$$

where

F_1 = horizontal pile forces along the structure axis (kips)

Q_1 = applied horizontal load in the U_1 direction (kips)

$$\Sigma F_1 = 8(-0.015) + 4(0.006) + 4(0.008) + 8(0.018) + 3(-0.0011) = 0$$

Similarly

$$\Sigma F_2 = Q_2$$

Table 18

Interactively Input Data for Example Problem 9

```
INPUT DATA FILE NAME IN 8 CHARACTERS OR LESS. HIT A
CARRIAGE RETURN IF INPUT DATA WILL COME FROM TERMINAL.
    ?




INPUT A FILE NAME FOR DATA.  HIT A CARRIAGE RETURN
IF YOU DO NOT WANT TO SAVE DATA FILE.
    ? DEB3


INPUT TWO LINES OF PROJECT IDENTIFICATION NOT
TO EXCEED 66 CHARACTERS EACH

INPUT FIRST LINE
    ? EXAMPLE PROBLEM NO. 9
INPUT SECOND LINE
    ? NCD EXAMPLE PROBLEM - CONSTANT SOIL MODULUS


DO YOU WANT TO RUN A 2-D OR 3-D ANALYSIS?
ENTER 2 OR 3 ? 3


INPUT NUMBER OF PILES, PILE GROUPS, AND LOADING CONDITIONS
    ? 27,1,1


INPUT SOIL PROPERTY DATA - MV AND ES:
      MV=1=CONSTANT SOIL OR MV=2=LINEARLY VARYING SOIL
      ES=SUBGRADE MODULUS (PSI IF MV=1 OR PCI IF MV=2)
    ? 1,200.0


DATA FOR PILE GROUP NO. -    1


INPUT PILE SHAPE DATA:
      NPA=IDENTIFICATION NUMBER OF FIRST PILE IN GROUP
      NPB=IDENTIFICATION NUMBER OF LAST PILE IN GROUP
      SLEN=LENGTH OF PILE (FEET)
      NES=CODE FOR TYPE OF INPUT TO COMPUTE ELASTIC PILE CONSTANTS
          1=INPUT PILE B MATRIX TERMS DIRECTLY
          2=ANY SHAPE PILE
          3=ROUND PILE
    ? 1,27,70.0,2


INPUT AIX & AIY-MOMENTS OF INERTIA (IN**4)
      AREA - CROSS SECTIONAL AREA (IN**2)
      X & Y - PILE DIMENSIONS ALONG X & Y AXES (INCHES)
    ? 1728.0,1728.0,144.0,12.0,12.0


INPUT PILE MATERIAL DATA-MP (1=CONCRETE, 2=TIMBER, 3=STEEL, 4=SPECIAL)
    ? 1


INPUT US=ULTIMATE STRENGTH OF CONCRETE (PSI)
      W=WEIGHT OF CONCRETE (PCF)
    ? 5000.0,150.0


INPUT FIXITY DATA - NF (1=INPUT ALL FIXITY COEFFICIENTS
                        OR 2=INPUT DEGREE OF FIXITY
    ? 2


INPUT DF - DEGREE OF FIXITY (0.0,0.5,1.0)
      PF - PILE RESISTANCE (1=BEARING OR 0.5=FRICTION)
      PFT - PARTICIPATION FACTOR FOR TORSION
      G - TORSION MODULUS (PSI)
    ? 0.0,1.0,0.0,0.0
```

(Continued)

Table 18 (Concluded)

```
INPUT ALLOWABLE LOADS AND MOMENTS:
      ACBT=ALLOWABLE AXIAL LOAD USED IN COMBINED BENDING
           FOR PILE IN TENSION (KIPS)
      AMINT=ALLOWABLE MOMENT ABOUT MINOR PRINCIPLE AXIS
           FOR PILE IN TENSION (KIP-FT)
      AMAJT=ALLOWABLE MOMENT ABOUT MAJOR PRINCIPLE AXIS
           FOR PILE IN TENSION (KIP-FT)
      ACBC=ALLOWABLE AXIAL LOAD USED IN COMBINED BENDING
           FOR PILE IN COMPRESSION (KIPS)
      AMINC=ALLOWABLE MOMENT ABOUT MINOR PRINCIPLE AXIS
           FOR PILE IN COMPRESSION (KIP-FT)
      AMAJC=ALLOWABLE MOMENT ABOUT MAJOR PRINCIPLE AXIS
           FOR PILE IN COMPRESSION (KIP-FT)
      ACL=ALLOWABLE COMPRESSIVE LOAD (KIPS)
      ATL=ALLOWABLE TENSILE LOAD (KIPS)
    ? 1000.0,1000.0,1000.0,1000.0,10[illegible]


INPUT IB: 0=INPUT BATTER FOR EACH PILE OR
      THE NUMBER OF SUBGROUPS WITH THE SAME BATTER
    ? 3


INPUT NFP-NO. OF FIRST PILE  NLP-NO. OF LAST PILE
      BATT-BATTER=BATT VERTICAL ON 1 HORIZONTAL
      ANGL=CLOCKWISE ANGLE BETWEEN POSITIVE X-AXIS OF THE
           STRUCTURE AND X-AXIS (DIRECTION OF BATTER) OF PILE (DEG)

FOR PILE SUBGROUP -    1 ? 1,12,3,270

FOR PILE SUBGROUP -    2 ? 13,24,3,90

FOR PILE SUBGROUP -    3 ? 25,27,0,0

THIS PROGRAM GENERATES THE FOLLOWING TABLES:

TABLE NO.                  CONTENTS
   1               PILE AND SOIL DATA
   2               PILE COORDINATES AND BATTER
   3               STIFFNESS AND FLEXIBILITY MATRICES FOR THE
                   STRUCTURE AND COORDINATES OF ELASTIC CENTER
   4               APPLIED LOADS
   5               STRUCTURE DEFLECTIONS
   6               PILE DEFLECTIONS ALONG PILE AXIS
   7               PILE FORCES ALONG PILE AXIS
   8               PILE FORCES ALONG STRUCTURE AXIS

INPUT THE NUMBERS OF THE TABLES FOR WHICH YOU WANT THE OUTPUT.
SEPARATE THE NUMBERS WITH COMMAS. ? 1,2,3,4,5,6,7,8


INPUT A FILENAME FOR TABLE 5 IN 8 CHARACTERS OR LESS
IF YOU WANT TO USE THIS INFORMATION FOR A NEW RUN
HIT A CARRIAGE RETURN IF YOU DO NOT WANT THIS FILE.
    ?


INPUT A FILE NAME FOR OUTPUT IN 8 CHARACTERS OR LESS.
HIT A CARRIAGE RETURN IF OUTPUT IS TO BE PRINTED ON TERMINAL.
    ?


INPUT U1'S - DISTANCES FOR ORIGIN TO PILE
      ALONG U1 AXIS
? 14,10,6,2,-2,-6,-10,-14,12,4,-4,-12,12,4,-4,-12,14,10,6,2,
? -2,-6,-10,-14,16,12,8


INPUT U2'S - DISTANCES FROM ORIGIN TO PILE
      ALONG U2 AXIS
? 8*1.5,4*4.5,4*9.0,8*12.0,3*3.0


INPUT U3'S - DISTANCES FROM ORIGIN TO PILE
      ALONG U3 AXIS
? 27*0.0


INPUT APPLIED LOADS AND MOMENTS:
      Q1 & Q2 - HORIZONTAL LOADS ALONG U1 & U2 AXES (KIPS)
      Q3 - VERTICAL LOAD ALONG U3 AXIS (KIPS)
      Q4,Q5,Q6 - MOMENTS ABOUT U1,U2,U3 AXES (KIP-FT)

FOR LOADING CONDITION -    1 ? 0.0 [illegible].961,[illegible]4.[illegible] [illegible] [illegible]
```

Table 19

Input Data for Example Problem 9

Group									
1A	10000	EXAMPLE PROBLEM NO. 9							
1B	10010	NOD EXAMPLE PROBLEM - CONSTANT SOIL MODULUS							
2A	10020	3							
2B	10030	27	1	1					
3	10040	1	200.000						
4A	10050	1	27	70.000	2				
4C	10060	1728.000	1728.000	144.00	12.000	12.000			
5A	10070	1							
5B	10080	5000.000	150.000						
6A	10090	2							
6C	10100	0.	1.200	0.	0.				
7	10110	1000.00	1000.00	1000.00	1000.00	1000.00	1000.00	1000.00	1000.00
8A	10120	0							
	10130	3.00	270.00	14.00	1.50	0.			
10	10140	3.00	270.00	10.00	1.50	0.			
	10150	3.00	270.00	6.00	1.50	0.			
	10160	3.00	270.00	2.00	1.50	0.			
	10170	3.00	270.00	-2.00	1.50	0.			
	10180	3.00	270.00	-6.00	1.50	0.			
	10190	3.00	270.00	-10.00	1.50	0.			
	10200	3.00	270.00	-14.00	1.50	0.			
	10210	3.00	270.00	12.00	4.50	0.			
	10220	3.00	270.00	4.00	4.50	0.			
	10230	3.00	270.00	-4.00	4.50	0.			
	10240	3.00	270.00	-12.00	4.50	0.			
	10250	3.00	90.00	12.00	9.00	0.			
	10260	3.00	90.00	4.00	9.00	0.			
	10270	3.00	90.00	-4.00	9.00	0.			
	10280	3.00	90.00	-12.00	9.00	0.			
	10290	3.00	90.00	14.00	12.00	0.			
	10300	3.00	90.00	10.00	12.00	0.			
	10310	3.00	90.00	6.00	12.00	0.			
	10320	3.00	90.00	2.00	12.00	0.			
	10330	3.00	90.00	-2.00	12.00	0.			
	10340	3.00	90.00	-6.00	12.00	0.			
	10350	3.00	90.00	-10.00	12.00	0.			
	10360	3.00	90.00	-14.00	12.00	0.			
	10370	0.	0.	16.00	3.00	0.			
	10380	0.	0.	12.00	3.00	0.			
	10390	0.	0.	8.00	3.00	0.			
11	10400	0.	275.961	344.900	5287.422	0.	0.		

Table 20

Output Data for Example Problem 9

```
EXAMPLE PROBLEM NO. 9
NOD EXAMPLE PROBLEM - CONSTANT SOIL MODULUS

NO. OF PILES =  27    B MATRIX IS CALCULATED FOR EACH PILE


************************************************************************


1. TABLE OF PILE AND SOIL DATA

PILE NUMBERS

   1  27  E = 0.43E 07 PSI    IX =  1728.00 IN**4    IY =  1728.00 IN**4
          AREA =  144.0 IN**2    X =     12.00 IN      Y =     12.00 IN
          LENGTH =    70.0 FEET    ES = 200.000
          K1 =    0.4107  K2 =    1.0000  K3 =    0.
          K4 =    0.        K5 =    0.        K6 =    0.


ALLOWABLES:  COMBINED BENDING FOR TENSION =  1000.000 KIPS
             MOMENT ABOUT MINOR AXIS FOR TENSION =  1000.000 KIP-FT
             MOMENT ABOUT MAJOR AXIS FOR TENSION =  1000.000 KIP-FT
             COMBINED BENDING FOR COMPRESSION =  1000.000 KIPS
             MOMENT ABOUT MINOR AXIS FOR COMPRESSION =  1000.000 KIP-FT
             MOMENT ABOUT MAJOR AXIS FOR COMPRESSION =  1000.000 KIP-FT
             COMPRESSIVE LOAD =  1000.000 KIPS
             TENSILE LOAD =  1000.000 KIPS

THE B MATRIX FOR PILES   1 THROUGH  27 IS

 0.110E 05  0.         0.         0.         0.         0.
 0.         0.110E 05  0.         0.         0.         0.
 0.         0.         0.735E 06  0.         0.         0.
 0.         0.         0.         0.         0.         0.
 0.         0.         0.         0.         0.         0.
 0.         0.         0.         0.         0.         0.


************************************************************************


2. TABLE OF PILE COORDINATES AND BATTER

PILE NO.  BATTER   ANGLE  U1(FT)   U2(FT)  U3(FT)
   1        3.00   270.   14.000   1.500   0.
   2        3.00   270.   10.000   1.500   0.
   3        3.00   270.    6.000   1.500   0.
   4        3.00   270.    2.000   1.500   0.
   5        3.00   270.   -2.000   1.500   0.
   6        3.00   270.   -6.000   1.500   0.
   7        3.00   270.  -10.000   1.500   0.
   8        3.00   270.  -14.000   1.500   0.
   9        3.00   270.   12.000   4.500   0.
  10        3.00   270.    4.000   4.500   0.
  11        3.00   270.   -4.000   4.500   0.
  12        3.00   270.  -12.000   4.500   0.
  13        3.00    90.   12.000   9.000   0.
  14        3.00    90.    4.000   9.000   0.
  15        3.00    90.   -4.000   9.000   0.
  16        3.00    90.  -12.000   9.000   0.
  17        3.00    90.   14.000  12.000   0.
  18        3.00    90.   10.000  12.000   0.
  19        3.00    90.    6.000  12.000   0.
  20        3.00    90.    2.000  12.000   0.
  21        3.00    90.   -2.000  12.000   0.
  22        3.00    90.   -6.000  12.000   0.
  23        3.00    90.  -10.000  12.000   0.
  24        3.00    90.  -14.000  12.000   0.
  25      VERTICAL   0.   16.000   3.000   0.
  26      VERTICAL   0.   12.000   3.000   0.
  27      VERTICAL   0.    8.000   3.000   0.


************************************************************************
```

(Continued)

(Sheet 1 of 3)

Table 20 (Continued)

3. STIFFNESS MATRIX S FOR THE STRUCTURE

0.298E 06	-0.949E-02	0.337E-01	0.127E 01	0.931E-09	-0.226E 08
-0.949E-02	0.204E 07	-0.234E-01	0.266E 09	0.150E 01	0.477E 07
0.337E-01	-0.234E-01	0.181E 08	0.137E 10	-0.317E 09	0.52[illegible] 00
0.127E 01	0.266E 09	0.137E 10	0.153E 12	-0.114E 11	0.256E 03
0.466E-09	0.500E 00	-0.317E 09	-0.114E 11	0.238E 12	0.64[illegible]E 03
-0.226E 08	0.477E 07	0.	0.256E 03	0.640E 03	0.271E 11

3A FLEXIBILITY MATRIX F FOR THE STRUCTURE

0.358E-05	-0.239E-07	-0.986E-08	0.129E-09	-0.695E-11	0.300E-08
-0.239E-07	0.167E-05	0.691E-06	-0.904E-08	0.487E-09	-0.314E-09
-0.986E-08	0.691E-06	0.461E-06	-0.529E-08	0.361E-09	-0.130E-09
0.129E-09	-0.904E-08	-0.529E-08	0.692E-10	-0.373E-11	0.170E-11
-0.695E-11	0.487E-09	0.361E-09	-0.373E-11	0.450E-11	-0.915E-13
0.300E-08	-0.314E-09	-0.130E-09	0.170E-11	-0.915E-13	0.394E-10

******** LOADING CONDITION 1 ********

****

4. MATRIX OF APPLIED LOADS Q (KIPS & FEET)

Q1	Q2	Q3	Q4	Q5	Q6
0.	276.961	344.902	5297.422	0.	0.

****

5. STRUCTURE DEFLECTIONS (INCHES)

D1	D2	D3	D4	D5	D6
-0.183E-02	0.128E 00	0.148E-01	0.614E-04	0.227E-04	-0.240E-04

****

6. PILE DEFLECTIONS ALONG PILE AXIS (INCHES)

PILE	X1	X2	X3	X4	X5	X6
1	-0.122E 00	-0.139E-02	-0.278E-01	-0.139E-04	0.614E-04	-0.300E-04
2	-0.123E 00	-0.139E-02	-0.271E-01	-0.139E-04	0.614E-04	-0.300E-04
3	-0.124E 00	-0.139E-02	-0.264E-01	-0.139E-04	0.614E-04	-0.300E-04
4	-0.126E 00	-0.139E-02	-0.258E-01	-0.139E-04	0.614E-04	-0.300E-04
5	-0.127E 00	-0.139E-02	-0.251E-01	-0.139E-04	0.614E-04	-0.300E-04
6	-0.129E 00	-0.139E-02	-0.244E-01	-0.139E-04	0.614E-04	-0.300E-04
7	-0.130E 00	-0.139E-02	-0.238E-01	-0.139E-04	0.614E-04	-0.300E-04
8	-0.132E 00	-0.139E-02	-0.231E-01	-0.139E-04	0.614E-04	-0.300E-04
9	-0.123E 00	-0.529E-03	-0.253E-01	-0.139E-04	0.614E-04	-0.300E-04
10	-0.126E 00	-0.529E-03	-0.240E-01	-0.139E-04	0.614E-04	-0.300E-04
11	-0.129E 00	-0.529E-03	-0.227E-01	-0.139E-04	0.614E-04	-0.300E-04
12	-0.132E 00	-0.529E-03	-0.213E-01	-0.139E-04	0.614E-04	-0.300E-04
13	0.113E 00	-0.770E-03	0.567E-01	0.291E-04	-0.614E-04	-0.156E-04
14	0.114E 00	-0.770E-03	0.595E-01	0.291E-04	-0.614E-04	-0.156E-04
15	0.116E 00	-0.770E-03	0.623E-01	0.291E-04	-0.614E-04	-0.156E-04
16	0.117E 00	-0.770E-03	0.651E-01	0.291E-04	-0.614E-04	-0.156E-04
17	0.111E 00	-0.164E-02	0.581E-01	0.291E-04	-0.614E-04	-0.156E-04
18	0.112E 00	-0.164E-02	0.595E-01	0.291E-04	-0.614E-04	-0.156E-04
19	0.113E 00	-0.164E-02	0.609E-01	0.291E-04	-0.614E-04	-0.156E-04
20	0.114E 00	-0.164E-02	0.623E-01	0.291E-04	-0.614E-04	-0.156E-04
21	0.114E 00	-0.164E-02	0.637E-01	0.291E-04	-0.614E-04	-0.156E-04
22	0.115E 00	-0.164E-02	0.651E-01	0.291E-04	-0.614E-04	-0.156E-04
23	0.116E 00	-0.164E-02	0.665E-01	0.291E-04	-0.614E-04	-0.156E-04
24	0.117E 00	-0.164E-02	0.678E-01	0.291E-04	-0.614E-04	-0.156E-04
25	-0.962E-03	0.124E 00	0.127E-01	0.614E-04	0.227E-04	-0.240E-04
26	-0.962E-03	0.125E 00	0.138E-01	0.614E-04	0.227E-04	-0.242E-04
27	-0.962E-03	0.126E 00	0.148E-01	0.614E-04	0.227E-04	-0.242E-04

(Continued)

(Sheet 2 of 3)

Table 20 (Concluded)

****

7. PILE FORCES ALONG PILE AXIS (KIPS & FEET)

PILE	F1	F2	F3	F4	F5	F6	CBFTH	FAILURE CB BI CC TE
1	-1.341	-0.015	-22.396	0.	0.	0.	0.02	
2	-1.357	-0.015	-19.905	0.	0.	0.	0.02	
3	-1.373	-0.015	-19.415	0.	0.	0.	0.02	
4	-1.389	-0.015	-18.925	0.	0.	0.	0.02	
5	-1.404	-0.015	-18.435	0.	0.	0.	0.02	
6	-1.420	-0.015	-17.944	0.	0.	0.	0.02	
7	-1.436	-0.015	-17.454	0.	0.	0.	0.02	
8	-1.452	-0.015	-16.964	0.	0.	0.	0.02	
9	-1.357	-0.006	-18.609	0.	0.	0.	0.02	
10	-1.388	-0.006	-17.628	0.	0.	0.	0.02	
11	-1.420	-0.006	-16.648	0.	0.	0.	0.02	
12	-1.452	-0.006	-15.667	0.	0.	0.	0.02	
13	1.241	-0.008	41.544	0.	0.	0.	0.04	
14	1.258	-0.008	43.697	0.	0.	0.	0.04	
15	1.274	-0.008	45.751	0.	0.	0.	0.05	
16	1.291	-0.005	47.305	0.	0.	0.	0.05	
17	1.229	-0.018	42.572	0.	0.	0.	0.04	
18	1.238	-0.019	43.699	0.	0.	0.	0.04	
19	1.246	-0.018	44.726	0.	0.	0.	0.04	
20	1.254	-0.018	45.753	0.	0.	0.	0.05	
21	1.263	-0.018	46.780	0.	0.	0.	0.05	
22	1.271	-0.018	47.805	0.	0.	0.	0.05	
23	1.279	-0.018	48.833	0.	0.	0.	0.05	
24	1.287	-0.018	49.860	0.	0.	0.	0.05	
25	-0.011	1.363	9.309	0.	0.	0.	0.01	
26	-0.011	1.375	10.109	0.	0.	0.	0.01	
27	-0.011	1.388	10.909	0.	0.	0.	0.01	

TOTAL NO. FAILURES = 0 LOAD CASE 1

****

8. PILE FORCES ALONG STRUCTURE AXIS (KIPS & FEET)

PILE	F1	F2	F3	F4	F5	F6
1	-0.015	7.722	-18.925	0.	0.	0.
2	-0.015	7.582	-18.455	0.	0.	0.
3	-0.015	7.442	-17.935	0.	0.	0.
4	-0.015	7.302	-17.515	0.	0.	0.
5	-0.015	7.162	-17.044	0.	0.	0.
6	-0.015	7.022	-16.574	0.	0.	0.
7	-0.015	6.882	-16.104	0.	0.	0.
8	-0.015	6.742	-15.634	0.	0.	0.
9	-0.006	7.172	-17.225	0.	0.	0.
10	-0.006	6.892	-16.285	0.	0.	0.
11	-0.006	6.612	-15.344	0.	0.	0.
12	-0.006	6.332	-14.404	0.	0.	0.
13	0.008	14.347	39.114	0.	0.	0.
14	0.006	15.012	41.057	0.	0.	0.
15	0.008	15.677	43.200	0.	0.	0.
16	0.008	16.342	44.943	0.	0.	0.
17	0.018	14.661	40.094	0.	0.	0.
18	0.018	14.993	41.065	0.	0.	0.
19	0.018	15.326	42.037	0.	0.	0.
20	0.018	15.658	43.008	0.	0.	0.
21	0.018	15.991	43.99[illegible]	0.	0.	0.
22	0.016	16.323	44.951	0.	0.	0.
23	0.018	16.656	45.923	0.	0.	0.
24	0.018	16.988	46.894	0.	0.	0.
25	-0.011	1.363	9.309	0.	0.	0.
26	-0.011	1.375	10.109	0.	0.	0.
27	-0.011	1.388	10.909	0.	0.	0.
SUM	-0.000	276.961	344.900	5287.422	0.000	-0.000

****

(Sheet 3 of 3)

$$\Sigma F_2 = 7.722 + 7.582 + 7.442 + 7.302 + 7.162 + 7.022$$
$$+ 6.882 + 6.742 + 7.172 + 6.892 + 6.612 + 6.332$$
$$+ 14.347 + 15.012 + 15.677 + 16.342 + 14.661$$
$$+ 14.993 + 15.326 + 15.658 + 15.991 + 16.323$$
$$+ 16.656 + 16.988 + 1.363 + 1.375 + 1.388$$
$$\Sigma F_2 = 277$$

and

$$\Sigma F_3 = Q_3$$

$$\Sigma F_3 = - 18.925 - 18.455 - 17.985 - 17.515 - 17.044$$
$$- 16.574 - 16.104 - 15.634 - 17.225 - 16.285$$
$$- 15.344 - 14.404 + 39.114 + 41.057 + 43.0$$
$$+ 44.943 + 40.094 + 41.065 + 42.037 + 43.008$$
$$+ 43.980 + 44.951 + 45.923 + 46.894 + 9.309$$
$$+ 10.109 + 10.909$$
$$\Sigma F_3 = 345$$

These results agree closely with the computer results (item 8).

Example Problem 10

Three-dimensional problem, 9 piles and linearly varying soil moduli

91. The tenth example problem illustrating the use of program LMVDPILE has linearly varying soil moduli and is taken from Saul (1968). Figures 23 and 24 show the physical problem. Figure 25 shows the properties and loading conditions. The input data are stored in a data file prior to running the program and are shown in Table 21. The computer output is presented in Table 22. This example illustrates how a three-dimensional problem with linearly varying soil moduli is coded. It also shows how battered piles are coded.

Results and calculations

92. From statics $\Sigma F = 0$. From the program output in Table 22

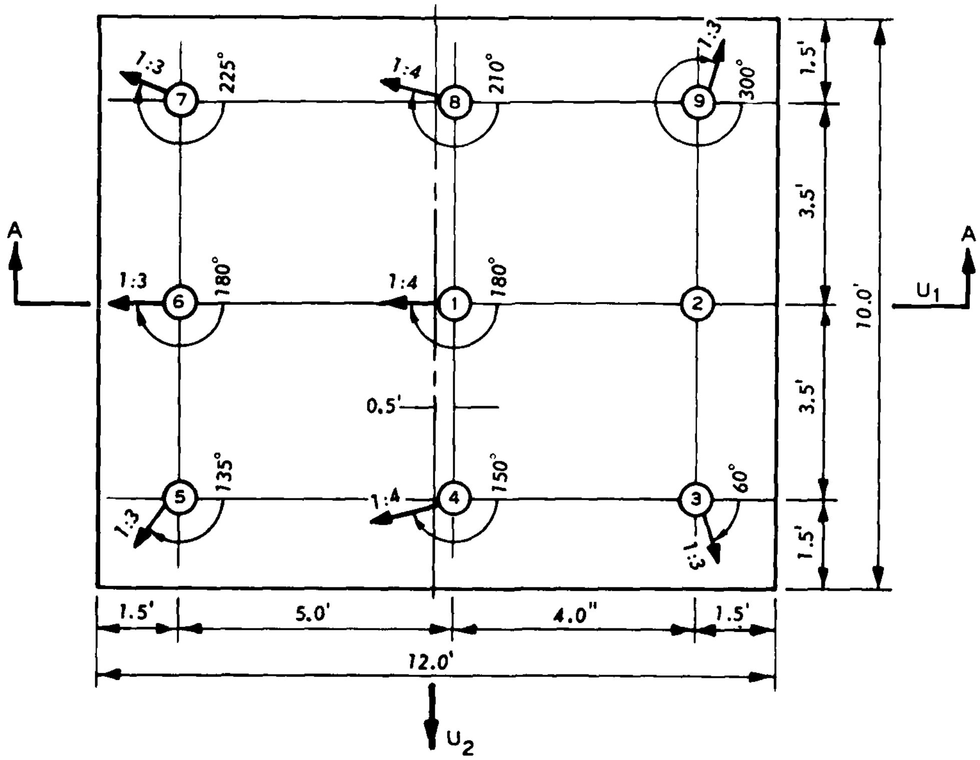

Figure 23. Plan view of example problem 10

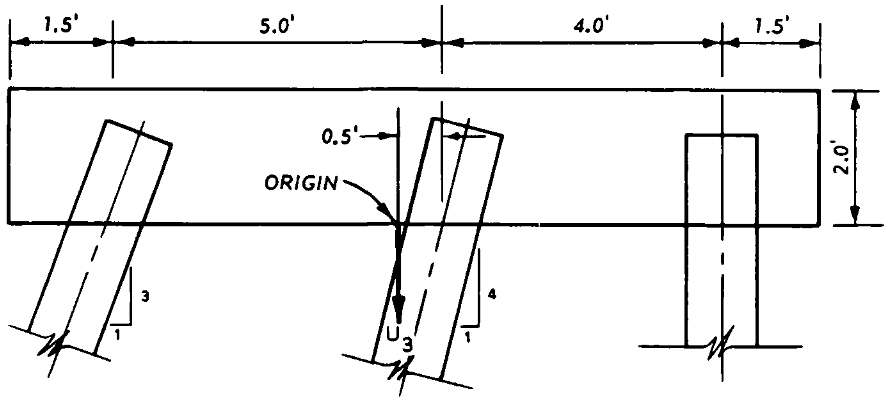

Figure 24. Section A-A for example problem 10

Properties	
$E = 0.3 \times 10^8$ psi	$K_1 = 0.576$
KS = 100.0 pci	$K_2 = 2.0$
$I_1 = 211.9$ in.^{4}	$K_3 = 1.043$
$I_2 = 211.9$ in.^{4}	$K_4 = 7063.3$
Area = 16.1 in.^{2}	$K_5 = 0.544$
Length = 120.0 ft	$K_6 = 0.544$

Loading Case	Q_1 (kips)	Q_2 (kips)	Q_3 (kips)	Q_4 (kip-ft)	Q_5 (kip-ft)	Q_6 (kip-ft)
1	200.0	100.0	1500.0	1000.0	4000.0	416.667

Figure 25. Properties and loading conditions for example problem 10

Table 21

Input Data for Example Problem 10

Group									
1A	10000	EXAMPLE PROBLEM NO. 10							
1B	10010	SLD CHECK PROBLEM NO. 2 - SAUL							
2A	10020	3							
2B	10030	9	1	1					
3A	10040	2	100.000						
4A	10050	1	9	120.000	2				
4C	10060	211.900	211.900	16.100	1.000	1.000			
5A	10070	4							
5C	10080	30000000.000							
6A	10090	1							
6B	10100	1.000	0.567	2.000	1.043	7063.300	0.544	0.544	
7	10110	500.00	333.33	333.33	500.00	333.33	333.33	600.00	100.00
8A	10120	2							
10	10130	4.000	150.000	0.500	0.	0.			
	10140	0.	0.	4.500	0.	0.			
	10150	3.000	60.000	4.500	3.500	0.			
	10160	4.000	150.000	0.500	3.500	0.			
	10170	3.000	135.000	-4.500	3.500	0.			
	10180	3.000	180.000	-4.500	0.	0.			
	10190	3.000	225.000	-4.500	-3.500	0.			
	10200	4.000	210.000	0.500	-3.500	0.			
	10210	3.000	300.000	4.500	-3.500	0.			
11	10220	200.000	100.000	1500.000	1000.000	4000.000	416.667		

Table 22

Output Data for Example Problem 10

```
EXAMPLE PROBLEM NO. 10
SLD CHECK PROBLEM NO. 2 - SAUL

NO. OF PILES =   9     B MATRIX IS CALCULATED FOR EACH PILE


************************************************************************

1. TABLE OF PILE AND SOIL DATA

PILE NUMBERS

   1   9  E = 0.30E 08 PSI    IX =    211.90 IN**4    IY =    211.90 IN**4
          AREA =   15.1 IN**2    X =       1.00 IN      Y =      1.00 IN
          LENGTH =  120.0 FEET    ES = 100.000
          K1 =    0.5670  K2 =   2.0000  K3 =   1.0430
          K4 =7063.3000  K5 =   0.5440  K6 =   0.5440


ALLOWABLES:  COMBINED BENDING FOR TENSION =    500.000 KIPS
             MOMENT ABOUT MINOR AXIS FOR TENSION =   333.330 KIP-FT
             MOMENT ABOUT MAJOR AXIS FOR TENSION =   333.330 KIP-FT
             COMBINED BENDING FOR COMPRESSION =   500.000 KIPS
             MOMENT ABOUT MINOR AXIS FOR COMPRESSION =   333.330 KIP-FT
             MOMENT ABOUT MAJOR AXIS FOR COMPRESSION =   333.330 KIP-FT
             COMPRESSIVE LOAD =    600.000 KIPS
             TENSILE LOAD =   100.000 KIPS

THE B MATRIX FOR PILES   1 THROUGH   9 IS

 0.750E 05  0.          0.          0.          0.262E 07  0.
 0.         0.750E 05   0.         -0.262E 07  0.          0.
 0.         0.          0.671E 06  0.          0.          0.
 0.        -0.262E 07   0.          0.182E 09  0.          0.
 0.262E 07  0.          0.          0.          0.182E 09  0.
 0.         0.          0.          0.          0.          0.706E 07


************************************************************************

2. TABLE OF PILE COORDINATES AND BATTER

PILE NO.  BATTER   ANGLE  U1(FT)  U2(FT)  U3(FT)
   1        4.00   180.   0.500   0.      0.
   2     VERTICAL   0.    4.500   0.      0.
   3        3.00    60.   4.500   3.500   0.
   4        4.00   150.   0.500   3.500   0.
   5        3.00   135.  -4.500   3.500   0.
   6        3.00   180.  -4.500   0.      0.
   7        3.00   225.  -4.500  -3.500   0.
   8        4.00   210.   0.500  -3.500   0.
   9        3.00   300.   4.500  -3.500   0.


************************************************************************

3. STIFFNESS MATRIX S FOR THE STRUCTURE

 0.911E 06  0.732E-03 -0.636E 06  0.313E 00 -0.803E 07  0.625E-01
 0.977E-03  0.841E 06  0.195E-02  0.698E 07  0.250E 00  0.177E 07
-0.636E 06  0.391E-02  0.563E 07  0.500E 00 -0.118E 08 -0.375E 00
 0.250E 00  0.698E 07  0.500E 00  0.940E 10  0.          0.857E 09
-0.803E 07  0.198E 00 -0.118E 08  0.160E 02  0.128E 11  0.280E 02
 0.156E 00  0.177E 07 -0.563E 00  0.857E 09  0.320E 02  0.305E 10

3A  FLEXIBILITY MATRIX F FOR THE STRUCTURE

 0.120E-05 -0.144E-14  0.137E-06 -0.534E-16  0.878E-09  0.513E-19
-0.158E-14  0.120E-05 -0.604E-15 -0.936E-09 -0.238E-16 -0.432E-09
 0.137E-06 -0.975E-15  0.194E-06 -0.198E-16  0.764E-09  0.244E-16
-0.392E-16 -0.936E-09 -0.183E-16  0.123E-09  0.622E-19 -0.341E-10
 0.878E-09 -0.172E-16  0.764E-09 -0.116E-18  0.788E-10 -0.667E-18
-0.335E-16 -0.432E-09  0.314E-16 -0.341E-10 -0.828E-18  0.338E-09
```

(Continued)

Table 22 (Concluded)

******** LOADING CONDITION 1 ********

****

4. MATRIX OF APPLIED LOADS Q (KIPS & FEET)

Q1	Q2	Q3	Q4	Q5	Q6
200.000	100.000	1500.000	1000.000	4000.000	416.667

****

5. STRUCTURE DEFLECTIONS (INCHES)

D1	D2	D3	D4	D5	D6
0.488E 00	0.106E 00	0.339E 00	0.122E-02	0.436E-02	0.124E-02

****

6. PILE DEFLECTIONS ALONG PILE AXIS (INCHES)

PILE	X1	X2	X3	X4	X5	X6
1	-0.548E 00	-0.114E 00	0.177E 00	-0.148E-02	-0.436E-02	0.905E-03
2	0.488E 00	0.173E 00	0.952E-01	0.122E-02	0.436E-02	0.124E-02
3	0.303E 00	-0.291E 00	0.255E 00	0.376E-02	0.113E-02	0.256E-02
4	-0.398E 00	-0.317E 00	0.267E 00	0.793E-03	-0.438E-02	0.147E-02
5	-0.461E 00	-0.337E 00	0.496E 00	0.172E-02	-0.394E-02	0.188E-02
6	-0.642E 00	-0.395E-01	0.382E 00	-0.154E-02	-0.436E-02	0.789E-03
7	-0.552E 00	0.354E 00	0.358E 00	-0.413E-02	-0.222E-02	-0.722E-04
8	-0.571E 00	0.172E 00	0.118E 00	-0.343E-02	-0.316E-02	0.417E-03
9	0.100E 00	0.554E 00	0.799E-01	-0.339E-02	0.323E-02	0.173E-03

****

7. PILE FORCES ALONG PILE AXIS (KIPS & FEET)

PILE	F1	F2	F3	F4	F5	F6	CBFTR	FAILURE CB	BU	CO	TE
1	-52.443	-4.660	118.577	2.319	-185.527	0.533	0.80				
2	48.003	9.801	63.888	-19.266	172.620	0.729	0.70				
3	25.657	-31.698	171.191	120.696	83.134	1.506	0.95				
4	-41.267	-25.815	179.017	81.059	-153.219	0.867	1.06	F			
5	-44.880	-29.717	332.913	99.416	-160.382	1.104	1.45	F			
6	-59.533	1.074	256.359	-14.844	-206.142	0.464	1.18	F			
7	-47.167	37.343	240.492	-139.903	-153.982	-0.043	1.36	F			
8	-51.049	21.848	79.427	-89.585	-172.434	0.245	0.94				
9	15.951	50.443	53.626	-172.408	70.896	0.102	0.84				

TOTAL NO. FAILURES = 4 LOAD CASE 1

****

8. PILE FORCES ALONG STRUCTURE AXIS (KIPS & FEET)

PILE	F1	F2	F3	F4	F5	F6
1	22.118	4.660	127.756	-2.379	185.527	-0.046
2	48.003	9.801	63.888	-19.266	172.620	0.729
3	66.681	52.119	154.292	-14.507	141.141	-36.739
4	9.977	24.049	183.681	8.324	172.116	-18.819
5	-23.322	65.349	330.021	46.470	180.344	-30.391
6	-24.589	-1.074	262.029	13.936	206.142	5.135
7	4.271	-48.542	243.066	-15.023	202.741	44.201
8	37.131	-3.791	89.437	-11.002	192.758	21.965
9	59.731	-2.570	45.830	-20.367	177.068	54.617
SUM	200.000	100.000	1500.000	1000.000	4000.000	416.667

****

this can be shown. For example, for a 200-kip applied horizontal load in the U_1 direction

$$\Sigma F_1 = Q_1$$

where

F_1 = horizontal pile force along the structure axis

Q_1 = applied horizontal load in the U_1 direction

$\Sigma F_1 = 22.12 + 48.00 + 66.7 + 9.98 - 23.32 - 24.59 + 4.27 + 37.13 + 59.73$

ΣF_1 = 200.0 kips

Similarly

$$\Sigma F_2 = Q_2$$

where

Q_2 = applied horizontal load in the U_2 direction

ΣF_2 = 100.0 kips

$\Sigma F_3 = Q_3$

where

F_3 = vertical pile force along the structure axis

Q_3 = applied vertical load in the U_3 direction

ΣF_3 = 1500.0 kips

These results agree with the computer program results.

93. Manual calculations for this example are presented in Saul's (1968) paper. The computer results presented in Table 22 agree closely with the classical method results. For example, a comparison of the moments about the U_1-axis (F4's) is shown below:

	F4 from	
Pile No.	Computer Output (kip-ft)	Saul's Example (kip-ft)
1	2.319	2.31
2	-19.266	-19.25
3	120.696	120.68

(Continued)

Pile No.	F4 from Computer Output (kip-ft)	F4 from Saul's Example (kip-ft)
4	81.059	81.03
5	99.416	99.38
6	-14.844	-14.85
7	-139.903	-139.88
8	-89.585	-89.58
9	-172.408	-172.36

Example Problem 11

Three-dimensional problem, 60 piles
with linearly varying soil moduli

94. This example problem is a three-dimensional system with 60 piles. The physical problem for this example is shown in Figure 26. The properties and loading conditions are shown in Figure 27. Table 23 shows the data file saved prior to the run. The computer output is presented in Table 24.

95. This example was run to verify that the computer results agree with the St. Louis District's program.

Results and calculations

96. The computer results shown in Table 24 agree closely with those from the St. Louis program output. For example, for pile 1 for load case 1, the pile forces along the structure axis from Table 24 are

$$F_1 = 42.305 \text{ kips}$$

$$F_2 = 0.0 \text{ kips}$$

$$F_3 = 120.806 \text{ kips}$$

$$F_4 = 0 \text{ kip-ft}$$

$$F_5 = 0 \text{ kip-ft}$$

$$F_6 = 0 \text{ kip-ft}$$

The St. Louis program produced

$$F_1 = 42.3 \text{ kips}$$

$$F_2 = 0.0 \text{ kips}$$

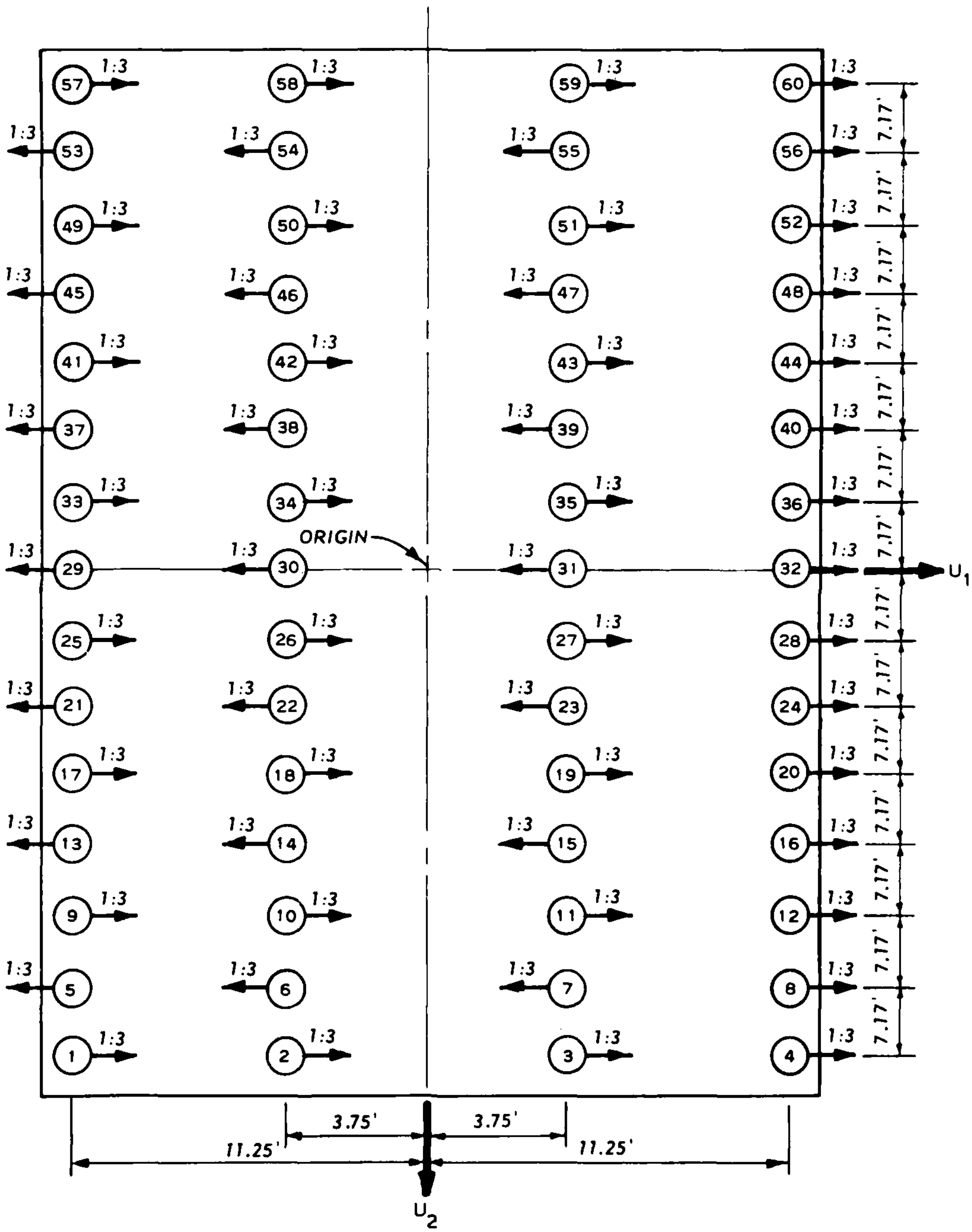

Figure 26. Physical problem for example problem 11

Properties	
$E = 0.3 \times 10^7$ psi	$K_1 = 0.411$
KS = 2.000 pci	$K_2 = 0.5$
$I_1 = 5461.333$ in.^{4}	K_3-$K_6 = 0.0$
$I_2 = 5461.333$ in.^{4}	
Area = 256.000 in.^{2}	
Length = 60.0 ft	

Loading Case	Q_1 (kips)	Q_2 (kips)	Q_3 (kips)	Q_4 (kip-ft)	Q_5 (kip-ft)	Q_6 (kip-ft)
1	1207.5	0.0	3113.25	0.0	4825.5875	0.0
2	1454.25	0.0	1683.15	0.0	-2743.5342	0.0
3	1825.95	0.0	875.5	0.0	-5779.62	0.0

Figure 27. Properties and loading conditions for example problem 11

$$F_3 = 120.8 \text{ kips}$$

$$F_4 = 0.0 \text{ kip-ft}$$

$$F_5 = 0.0 \text{ kip-ft}$$

$$F_6 = 0.0 \text{ kip-ft}$$

These results agree very closely.

Table 23

Input Data for Example Problem 11

Group									
1A	10000	EXAMPLF PROBLEM NO. 11							
1B	10010	TECHE-VERMILION OUTLET STRUCTURE 7.17 FT SPACING							
2A	10020	3							
2B	10030	60	1	3					
3	10040	2	2.000						
4A	10050	1	60	50	2				
4C	10060	5461.333	5461.333	256.000	16.000	16.000			
5A	10070	4							
5C	10080	3000000.000							
6A	10090	1							
6B	10100	0.	0.411	0.500	0.	0.	0.	0.	
7	10110	268.80	59.70	59.70	268.8	59.70	59.70	150.0	100.70
8A	10120	0							
10	10130	3.000	0.	-11.250	50.170	0.			
	10140	3.000	0.	-3.750	50.170	0.			
	10150	3.000	0.	3.750	50.170	0.			
	10160	3.000	0.	11.250	50.170	0.			
	10170	3.000	180.000	-11.250	43.000	0.			
	10180	3.000	180.000	-3.750	43.000	0.			
	10190	3.000	180.000	3.750	43.000	0.			
	10200	3.000	0.	11.250	43.000	0.			
	10210	3.000	0.	-11.250	35.833	0.			
	10220	3.000	0.	-3.750	35.833	0.			
	10230	3.000	0.	3.750	35.833	0.			
	10240	3.000	0.	11.250	35.833	0.			
	10250	3.000	180.000	-11.250	28.667	0.			
	10260	3.000	180.000	-3.750	28.667	0.			
	10270	3.000	180.000	3.750	28.667	0.			
	10280	3.000	0.	11.250	28.667	0.			
	10290	3.000	0.	-11.25	21.500	0.			
	10300	3.000	0.	-3.750	21.500	0.			
	10310	3.000	0.	3.750	21.500	0.			
	10320	3.000	0.	11.250	21.500	0.			
	10330	3.000	180.000	-11.250	14.333	0.			
	10340	3.000	180.000	-3.750	14.333	0.			
	10350	3.000	180.000	3.750	14.333	0.			
	10360	3.000	0.	11.250	14.333	0.			
	10370	3.000	0.	-11.250	7.170	0.			
	10380	3.000	0.	-3.750	7.170	0.			
	10390	3.000	0.	3.750	7.170	0.			
	10400	3.000	0.	11.250	7.170	0.			
	10410	3.000	180.000	-11.250	0.	0.			
	10420	3.000	180.000	-3.750	0.	0.			
	10430	3.000	180.000	3.750	0.	0.			
	10440	3.000	0.	11.250	0.	0.			
	10450	3.000	0.	-11.250	-7.170	0.			
	10460	3.000	0.	-3.750	-7.170	0.			
	10470	3.000	0.	3.750	-7.170	0.			
	10480	3.000	0.	11.250	-7.170	0.			
	10490	3.000	180.000	-11.250	-14.333	0.			
	10500	3.000	180.000	-3.750	-14.333	0.			
	10510	3.000	180.000	3.750	-14.333	0.			
	10520	3.000	0.	11.250	-14.333	0.			
	10530	3.000	0.	-11.250	-21.500	0.			
	10540	3.000	0.	-3.750	-21.500	0.			
	10550	3.000	0.	3.750	-21.500	0.			
	10560	3.000	0.	11.250	-21.500	0.			
	10570	3.000	180.000	-11.250	-28.667	0.			
	10580	3.000	180.000	-3.750	-28.667	0.			
	10590	3.000	180.000	3.750	-28.667	0.			
	10600	3.000	0.	11.250	-28.667	0.			
	10610	3.000	0.	-11.250	-35.833	0.			
	10620	3.000	0.	-3.750	-35.833	0.			
	10630	3.000	0.	3.750	-35.833	0.			
	10640	3.000	0.	11.250	-35.833	0.			
	10650	3.000	180.000	-11.250	-43.000	0.			
	10660	3.000	180.000	-3.750	-43.000	0.			
	10670	3.000	180.000	3.750	-43.000	0.			
	10680	3.000	0.	11.250	-43.000	0.			
	10690	3.000	0.	-11.250	-50.17	0.			
	10700	3.000	0.	-3.750	-50.170	0.			
	10710	3.000	0.	3.750	-50.170	0.			
	10720	3.000	0.	11.250	-50.170	0.			
	10730	1207.500	0.	3113.250	0.	4825.5875	0.		
11	10740	1454.250	0.	1683.150	0.	-2743.5342	0.		
	10750	1825.950	0.	875.500	0.	-5779.620	0.		

Table 24

Output Data for Example Problem 11

```
EXAMPLE PROBLEM NO. 11
TECHE-VERMILION OUTLET STRUCTURE  7.17 FT SPACING

NO. OF PILES =  60     B MATRIX IS CALCULATED FOR EACH PILE


************************************************************************


1. TABLE OF PILE AND SOIL DATA

PILE NUMBERS

   1  60  E = 0.30E 07 PSI    IX =  5461.33 IN**4    IY =  5461.33 IN**4
          AREA =  256.0 IN**2    X =     16.00 IN      Y =    16.00 IN
          LENGTH =   50.0 FEET    ES =   2.000
          K1 =   0.4110  K2 =   0.5000  K3 =   0.
          K4 =   0.      K5 =   0.      K6 =   0.


ALLOWABLES:  COMBINED BENDING FOR TENSION =   268.800 KIPS
             MOMENT ABOUT MINOR AXIS FOR TENSION =    59.700 KIP-FT
             MOMENT ABOUT MAJOR AXIS FOR TENSION =    59.700 KIP-FT
             COMBINED BENDING FOR COMPRESSION =   268.800 KIPS
             MOMENT ABOUT MINOR AXIS FOR COMPRESSION =    59.700 KIP-FT
             MOMENT ABOUT MAJOR AXIS FOR COMPRESSION =    59.700 KIP-FT
             COMPRESSIVE LOAD =   150.000 KIPS
             TENSILE LOAD =   100.000 KIPS

THE B MATRIX FOR PILES   1 THROUGH  60 IS

 0.759E 04  0.         0.         0.         0.         0.
 0.         0.759E 04  0.         0.         0.         0.
 0.         0.         0.533E 06  0.         0.         0.
 0.         0.         0.         0.         0.         0.
 0.         0.         0.         0.         0.         0.
 0.         0.         0.         0.         0.         0.


************************************************************************
```

(Continued)

(Sheet 1 of 11)

Table 24 (Continued)

2. TABLE OF PILE COORDINATES AND BATTER

PILE NO.	BATTER	ANGLE	U1(FT)	U2(FT)	U3(FT)
1	3.00	0.	-11.250	50.170	0.
2	3.00	0.	-3.750	50.170	0.
3	3.00	0.	3.750	50.170	0.
4	3.00	0.	11.250	50.170	0.
5	3.00	180.	-11.250	43.000	0.
6	3.00	180.	-3.750	43.000	0.
7	3.00	180.	3.750	43.000	0.
8	3.00	0.	11.250	43.000	0.
9	3.00	0.	-11.250	35.833	0.
10	3.00	0.	-3.750	35.833	0.
11	3.00	0.	3.750	35.833	0.
12	3.00	0.	11.250	35.833	0.
13	3.00	180.	-11.250	28.667	0.
14	3.00	180.	-3.750	28.667	0.
15	3.00	180.	3.750	28.667	0.
16	3.00	0.	11.250	28.667	0.
17	3.00	0.	-11.250	21.500	0.
18	3.00	0.	-3.750	21.500	0.
19	3.00	0.	3.750	21.500	0.
20	3.00	0.	11.250	21.500	0.
21	3.00	180.	-11.250	14.333	0.
22	3.00	180.	-3.750	14.333	0.
23	3.00	180.	3.750	14.333	0.
24	3.00	0.	11.250	14.333	0.
25	3.00	0.	-11.250	7.170	0.
26	3.00	0.	-3.750	7.170	0.
27	3.00	0.	3.750	7.170	0.
28	3.00	0.	11.250	7.170	0.
29	3.00	180.	-11.250	0.	0.
30	3.00	180.	-3.750	0.	0.
31	3.00	180.	3.750	0.	0.
32	3.00	0.	11.250	0.	0.
33	3.00	0.	-11.250	-7.170	0.
34	3.00	0.	-3.750	-7.170	0.
35	3.00	0.	3.750	-7.170	0.
36	3.00	0.	11.250	-7.170	0.
37	3.00	180.	-11.250	-14.333	0.
38	3.00	180.	-3.750	-14.333	0.
39	3.00	180.	3.750	-14.333	0.
40	3.00	0.	11.250	-14.333	0.
41	3.00	0.	-11.250	-21.500	0.
42	3.00	0.	-3.750	-21.500	0.
43	3.00	0.	3.750	-21.500	0.
44	3.00	0.	11.250	-21.500	0.
45	3.00	180.	-11.250	-28.667	0.
46	3.00	180.	-3.750	-28.667	0.
47	3.00	180.	3.750	-28.667	0.
48	3.00	0.	11.250	-28.667	0.
49	3.00	0.	-11.250	-35.833	0.
50	3.00	0.	-3.750	-35.833	0.
51	3.00	0.	3.750	-35.833	0.
52	3.00	0.	11.250	-35.833	0.
53	3.00	180.	-11.250	-43.000	0.
54	3.00	180.	-3.750	-43.000	0.
55	3.00	180.	3.750	-43.000	0.
56	3.00	0.	11.250	-43.000	0.
57	3.00	0.	-11.250	-50.170	0.
58	3.00	0.	-3.750	-50.170	0.
59	3.00	0.	3.750	-50.170	0.
60	3.00	0.	11.250	-50.170	0.

****

3. STIFFNESS MATRIX S FOR THE STRUCTURE

0.361E 07	-0.219E-02	0.284E 07	0.160E 02	-0.298E 09	0.700E 01
-0.219E-02	0.455E 06	0.657E-02	0.180E-07	0.296E 00	0.547E-01
0.284E 07	0.657E-02	0.238E 08	0.320E 02	0.500E 00	0.800E 01
0.160E 02	0.180E-07	0.320E 02	0.598E 13	0.512E 03	-0.523E 12
-0.298E 09	0.296E 00	0.	0.102E 04	0.242E 12	0.128E 04
0.	0.859E-01	0.800E 01	-0.523E 12	0.540E 03	0.503E 12

3A FLEXIBILITY MATRIX F FOR THE STRUCTURE

0.330E-06	0.184E-14	-0.325E-07	0.203E-17	0.337E-09	-0.711E-17
0.184E-14	0.220E-05	-0.661E-15	-0.803E-19	-0.346E-18	-0.270E-18
-0.325E-07	-0.661E-15	0.379E-07	-0.201E-16	-0.332E-10	-0.274E-18
-0.127E-17	-0.570E-19	-0.201E-18	0.291E-12	-0.247E-20	0.302E-12
0.337E-09	-0.346E-18	-0.332E-10	-0.447E-20	0.577E-11	-0.184E-19
-0.123E-17	-0.434E-18	-0.852E-18	0.302E-12	-0.683E-20	0.230E-11

(Continued)

(Sheet 2 of 11)

Table 24 (Continued)

******** LOADING CONDITION 1 ********

****

4. MATRIX OF APPLIED LOADS Q (KIPS & FEET)

Q1	Q2	Q3	Q4	Q5	Q6
1207.500	0.	3113.250	0.	4825.588	0.

****

5. STRUCTURE DEFLECTIONS (INCHES)

D1	D2	D3	D4	D5	D6
0.317E 00	0.804E-10	0.767E-01	-0.340E-11	0.522E-03	-0.105E-10

****

6. PILE DEFLECTIONS ALONG PILE AXIS (INCHES)

PILE	X1	X2	X3	X4	X5	X6
1	0.254E 00	0.150E-08	0.240E 00	0.919E-13	0.522E-03	-0.110E-10
2	0.269E 00	0.553E-09	0.195E 00	0.919E-13	0.522E-03	-0.110E-10
3	0.284E 00	-0.392E-09	0.151E 00	0.919E-13	0.522E-03	-0.110E-10
4	0.299E 00	-0.134E-08	0.106E 00	0.919E-13	0.522E-03	-0.110E-10
5	-0.348E 00	-0.213E-08	0.393E-01	0.753E-11	-0.522E-03	-0.856E-11
6	-0.333E 00	-0.118E-08	-0.528E-02	0.753E-11	-0.522E-03	-0.856E-11
7	-0.318E 00	-0.238E-09	-0.499E-01	0.753E-11	-0.522E-03	-0.856E-11
8	0.299E 00	-0.134E-08	0.106E 00	0.919E-13	0.522E-03	-0.110E-10
9	0.254E 00	0.150E-08	0.240E 00	0.919E-13	0.522E-03	-0.110E-10
10	0.269E 00	0.553E-09	0.195E 00	0.919E-13	0.522E-03	-0.110E-10
11	0.284E 00	-0.392E-09	0.151E 00	0.919E-13	0.522E-03	-0.110E-10
12	0.299E 00	-0.134E-08	0.106E 00	0.919E-13	0.522E-03	-0.110E-10
13	-0.348E 00	-0.213E-08	0.393E-01	0.753E-11	-0.522E-03	-0.856E-11
14	-0.333E 00	-0.118E-08	-0.528E-02	0.753E-11	-0.522E-03	-0.856E-11
15	-0.318E 00	-0.238E-09	-0.499E-01	0.753E-11	-0.522E-03	-0.856E-11
16	0.299E 00	-0.134E-08	0.106E 00	0.919E-13	0.522E-03	-0.110E-10
17	0.254E 00	0.150E-08	0.240E 00	0.919E-13	0.522E-03	-0.110E-10
18	0.269E 00	0.553E-09	0.195E 00	0.919E-13	0.522E-03	-0.110E-10
19	0.284E 00	-0.392E-09	0.151E 00	0.919E-13	0.522E-03	-0.110E-10
20	0.299E 00	-0.134E-08	0.106E 00	0.919E-13	0.522E-03	-0.110E-10
21	-0.348E 00	-0.213E-08	0.393E-01	0.753E-11	-0.522E-03	-0.856E-11
22	-0.333E 00	-0.118E-08	-0.528E-02	0.753E-11	-0.522E-03	-0.856E-11
23	-0.318E 00	-0.238E-09	-0.499E-01	0.753E-11	-0.522E-03	-0.856E-11
24	0.299E 00	-0.134E-08	0.106E 00	0.919E-13	0.522E-03	-0.110E-10
25	0.254E 00	0.150E-08	0.240E 00	0.919E-13	0.522E-03	-0.110E-10
26	0.269E 00	0.553E-09	0.195E 00	0.919E-13	0.522E-03	-0.110E-10
27	0.284E 00	-0.392E-09	0.151E 00	0.919E-13	0.522E-03	-0.110E-10
28	0.299E 00	-0.134E-08	0.106E 00	0.919E-13	0.522E-03	-0.110E-10
29	-0.348E 00	-0.213E-08	0.393E-01	0.753E-11	-0.522E-03	-0.856E-11
30	-0.333E 00	-0.118E-08	-0.528E-02	0.753E-11	-0.522E-03	-0.856E-11
31	-0.318E 00	-0.238E-09	-0.499E-01	0.753E-11	-0.522E-03	-0.856E-11
32	0.299E 00	-0.134E-08	0.106E 00	0.919E-13	0.522E-03	-0.110E-10
33	0.254E 00	0.150E-08	0.240E 00	0.919E-13	0.522E-03	-0.110E-10
34	0.269E 00	0.553E-09	0.195E 00	0.919E-13	0.522E-03	-0.110E-10
35	0.284E 00	-0.392E-09	0.151E 00	0.919E-13	0.522E-03	-0.110E-10
36	0.299E 00	-0.134E-08	0.106E 00	0.919E-13	0.522E-03	-0.110E-10
37	-0.348E 00	-0.213E-08	0.393E-01	0.753E-11	-0.522E-03	-0.856E-11
38	-0.333E 00	-0.118E-08	-0.528E-02	0.753E-11	-0.522E-03	-0.856E-11
39	-0.318E 00	-0.238E-09	-0.499E-01	0.753E-11	-0.522E-03	-0.856E-11
40	0.299E 00	-0.134E-08	0.106E 00	0.919E-13	0.522E-03	-0.110E-10
41	0.254E 00	0.150E-08	0.240E 00	0.919E-13	0.522E-03	-0.110E-10
42	0.269E 00	0.553E-09	0.195E 00	0.919E-13	0.522E-03	-0.110E-10
43	0.284E 00	-0.392E-09	0.151E 00	0.919E-13	0.522E-03	-0.110E-10
44	0.299E 00	-0.134E-08	0.106E 00	0.919E-13	0.522E-03	-0.110E-10
45	-0.348E 00	-0.213E-08	0.393E-01	0.753E-11	-0.522E-03	-0.856E-11
46	-0.333E 00	-0.118E-08	-0.528E-02	0.753E-11	-0.522E-03	-0.856E-11
47	-0.318E 00	-0.238E-09	-0.499E-01	0.753E-11	-0.522E-03	-0.856E-11
48	0.299E 00	-0.134E-08	0.106E 00	0.919E-13	0.522E-03	-0.110E-10
49	0.254E 00	0.150E-08	0.240E 00	0.919E-13	0.522E-03	-0.110E-10
50	0.269E 00	0.553E-09	0.195E 00	0.919E-13	0.522E-03	-0.110E-10
51	0.284E 00	-0.392E-09	0.151E 00	0.919E-13	0.522E-03	-0.110E-10
52	0.299E 00	-0.134E-08	0.106E 00	0.919E-13	0.522E-03	-0.110E-10
53	-0.348E 00	-0.213E-08	0.393E-01	0.753E-11	-0.522E-03	-0.856E-11
54	-0.333E 00	-0.118E-08	-0.528E-02	0.753E-11	-0.522E-03	-0.856E-11
55	-0.318E 00	-0.238E-09	-0.499E-01	0.753E-11	-0.522E-03	-0.856E-11
56	0.299E 00	-0.134E-08	0.106E 00	0.919E-13	0.522E-03	-0.110E-10
57	0.254E 00	0.150E-08	0.240E 00	0.919E-13	0.522E-03	-0.110E-10
58	0.269E 00	0.553E-09	0.195E 00	0.919E-13	0.522E-03	-0.110E-10
59	0.284E 00	-0.392E-09	0.151E 00	0.919E-13	0.522E-03	-0.110E-10
60	0.299E 00	-0.134E-08	0.106E 00	0.919E-13	0.522E-03	-0.110E-10

(Continued)

(Sheet 3 of 11)

Table 24 (Continued)

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7. PILE FORCES ALONG PILE AXIS (KIPS & FEET)

PILE	F1	F2	F3	F4	F5	F6	CBFTR	FAILURE CB BU CO TE
1	1.931	0.000	127.985	0.	0.	0.	0.48	
2	2.044	0.000	104.209	0.	0.	0.	0.39	
3	2.157	-0.000	80.433	0.	0.	0.	0.30	
4	2.270	-0.000	56.657	0.	0.	0.	0.21	
5	-2.638	-0.000	20.957	0.	0.	0.	0.08	
6	-2.525	-0.000	-2.818	0.	0.	0.	0.01	
7	-2.412	-0.000	-26.594	0.	0.	0.	0.10	
8	2.270	-0.000	56.657	0.	0.	0.	0.21	
9	1.931	0.000	127.985	0.	0.	0.	0.48	
10	2.044	0.000	104.209	0.	0.	0.	0.39	
11	2.157	-0.000	80.433	0.	0.	0.	0.30	
12	2.270	-0.000	56.657	0.	0.	0.	0.21	
13	-2.638	-0.000	20.957	0.	0.	0.	0.08	
14	-2.525	-0.000	-2.818	0.	0.	0.	0.01	
15	-2.412	-0.000	-26.594	0.	0.	0.	0.10	
16	2.270	-0.000	56.657	0.	0.	0.	0.21	
17	1.931	0.000	127.985	0.	0.	0.	0.48	
18	2.044	0.000	104.209	0.	0.	0.	0.39	
19	2.157	-0.000	80.433	0.	0.	0.	0.30	
20	2.270	-0.000	56.657	0.	0.	0.	0.21	
21	-2.638	-0.000	20.957	0.	0.	0.	0.08	
22	-2.525	-0.000	-2.818	0.	0.	0.	0.01	
23	-2.412	-0.000	-26.594	0.	0.	0.	0.10	
24	2.270	-0.000	56.657	0.	0.	0.	0.21	
25	1.931	0.000	127.985	0.	0.	0.	0.48	
26	2.044	0.000	104.209	0.	0.	0.	0.39	
27	2.157	-0.000	80.433	0.	0.	0.	0.30	
28	2.270	-0.000	56.657	0.	0.	0.	0.21	
29	-2.638	-0.000	20.957	0.	0.	0.	0.08	
30	-2.525	-0.000	-2.818	0.	0.	0.	0.01	
31	-2.412	-0.000	-26.594	0.	0.	0.	0.10	
32	2.270	-0.000	56.657	0.	0.	0.	0.21	
33	1.931	0.000	127.985	0.	0.	0.	0.48	
34	2.044	0.000	104.209	0.	0.	0.	0.39	
35	2.157	-0.000	80.433	0.	0.	0.	0.30	
36	2.270	-0.000	56.657	0.	0.	0.	0.21	
37	-2.638	-0.000	20.957	0.	0.	0.	0.08	
38	-2.525	-0.000	-2.818	0.	0.	0.	0.01	
39	-2.412	-0.000	-26.594	0.	0.	0.	0.10	
40	2.270	-0.000	56.657	0.	0.	0.	0.21	
41	1.931	0.000	127.985	0.	0.	0.	0.48	
42	2.044	0.000	104.209	0.	0.	0.	0.39	
43	2.157	-0.000	80.433	0.	0.	0.	0.30	
44	2.270	-0.000	56.657	0.	0.	0.	0.21	
45	-2.638	-0.000	20.957	0.	0.	0.	0.08	
46	-2.525	-0.000	-2.818	0.	0.	0.	0.01	
47	-2.412	-0.000	-26.594	0.	0.	0.	0.10	
48	2.270	-0.000	56.657	0.	0.	0.	0.21	
49	1.931	0.000	127.985	0.	0.	0.	0.48	
50	2.044	0.000	104.209	0.	0.	0.	0.39	
51	2.157	-0.000	80.433	0.	0.	0.	0.30	
52	2.270	-0.000	56.657	0.	0.	0.	0.21	
53	-2.638	-0.000	20.957	0.	0.	0.	0.08	
54	-2.525	-0.000	-2.818	0.	0.	0.	0.01	
55	-2.412	-0.000	-26.594	0.	0.	0.	0.10	
56	2.270	-0.000	56.657	0.	0.	0.	0.21	
57	1.931	0.000	127.985	0.	0.	0.	0.48	
58	2.044	0.000	104.209	0.	0.	0.	0.39	
59	2.157	-0.000	80.433	0.	0.	0.	0.30	
60	2.270	-0.000	56.657	0.	0.	0.	0.21	

TOTAL NO. FAILURES = 0 LOAD CASE 1

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(Continued)

(Sheet 4 of 11)

Table 24 (Continued)

e. PILE FORCES ALONG STRUCTURE AXIS (KIPS - FEET)

PILE	F1	F2	F3	F4	F5	F6
1	42.305	0.000	120.806	0.	0.	0.
2	34.893	0.000	98.215	0.	0.	0.
3	27.481	-0.000	75.623	0.	0.	0.
4	20.070	-0.000	53.032	0.	0.	0.
5	-4.125	0.000	20.716	0.	0.	0.
6	3.287	0.000	-1.875	0.	0.	0.
7	10.698	-0.000	-24.467	0.	0.	0.
8	20.070	-0.000	53.032	0.	0.	0.
9	42.305	0.000	120.806	0.	0.	0.
10	34.893	0.000	98.215	0.	0.	0.
11	27.481	-0.000	75.623	0.	0.	0.
12	20.070	-0.000	53.032	0.	0.	0.
13	-4.125	0.000	20.716	0.	0.	0.
14	3.287	0.000	-1.875	0.	0.	0.
15	10.698	-0.000	-24.467	0.	0.	0.
16	20.070	-0.000	53.032	0.	0.	0.
17	42.305	0.000	120.806	0.	0.	0.
18	34.893	0.000	98.215	0.	0.	0.
19	27.481	-0.000	75.623	0.	0.	0.
20	20.070	-0.000	53.032	0.	0.	0.
21	-4.125	0.000	20.716	0.	0.	0.
22	3.287	0.000	-1.875	0.	0.	0.
23	10.698	-0.000	-24.467	0.	0.	0.
24	20.070	-0.000	53.032	0.	0.	0.
25	42.305	0.000	120.806	0.	0.	0.
26	34.893	0.000	98.215	0.	0.	0.
27	27.481	-0.000	75.623	0.	0.	0.
28	20.070	-0.000	53.032	0.	0.	0.
29	-4.125	0.000	20.716	0.	0.	0.
30	3.287	0.000	-1.875	0.	0.	0.
31	10.698	-0.000	-24.467	0.	0.	0.
32	20.070	-0.000	53.032	0.	0.	0.
33	42.305	0.000	120.806	0.	0.	0.
34	34.893	0.000	98.215	0.	0.	0.
35	27.481	-0.000	75.623	0.	0.	0.
36	20.070	-0.000	53.032	0.	0.	0.
37	-4.125	0.000	20.716	0.	0.	0.
38	3.287	0.000	-1.875	0.	0.	0.
39	10.698	-0.000	-24.467	0.	0.	0.
40	20.070	-0.000	53.032	0.	0.	0.
41	42.305	0.000	120.805	0.	0.	0.
42	34.893	0.000	98.215	0.	0.	0.
43	27.481	-0.000	75.623	0.	0.	0.
44	20.070	-0.000	53.032	0.	0.	0.
45	-4.125	0.000	20.716	0.	0.	0.
46	3.287	0.000	-1.875	0.	0.	0.
47	10.698	-0.000	-24.467	0.	0.	0.
48	20.070	-0.000	53.032	0.	0.	0.
49	42.305	0.000	120.806	0.	0.	0.
50	34.893	0.000	98.215	0.	0.	0.
51	27.481	-0.000	75.623	0.	0.	0.
52	20.070	-0.000	53.032	0.	0.	0.
53	-4.125	0.000	20.716	0.	0.	0.
54	3.287	0.000	-1.875	0.	0.	0.
55	10.698	-0.000	-24.467	0.	0.	0.
56	20.070	-0.000	53.032	0.	0.	0.
57	42.305	0.000	120.806	0.	0.	0.
58	34.893	0.000	98.215	0.	0.	0.
59	27.481	-0.000	75.623	0.	0.	0.
60	20.070	-0.000	53.032	0.	0.	0.
SUM	1207.500	0.000	3113.250	-0.001	4825.587	-0.000

(Continued)

(Sheet 5 of 11)

******** LOADING CONDITION 2 ********

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4. MATRIX OF APPLIED LOADS Q (KIPS & FEET)

Q1	Q2	Q3	Q4	Q5	Q6
1454.250	0.	1683.150	0.	-2743.534	0.

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5. STRUCTURE DEFLECTIONS (INCHES)

D1	D2	D3	D4	D5	D6
0.415E 00	0.154E-08	0.175E-01	-0.322E-11	0.311E-03	-0.102E-10

****

6. PILE DEFLECTIONS ALONG PILE AXIS (INCHES)

PILE	X1	X2	X3	X4	X5	X6
1	0.375E 00	0.292E-08	0.188E 00	0.164E-12	0.311E-03	-0.107E-10
2	0.383E 00	0.200E-08	0.161E 00	0.164E-12	0.311E-03	-0.107E-10
3	0.392E 00	0.108E-08	0.135E 00	0.164E-12	0.311E-03	-0.107E-10
4	0.401E 00	0.165E-09	0.108E 00	0.164E-12	0.311E-03	-0.107E-10
5	-0.412E 00	-0.374E-08	-0.747E-01	0.687E-11	-0.311E-03	-0.845E-11
6	-0.403E 00	-0.282E-08	-0.101E 00	0.687E-11	-0.311E-03	-0.845E-11
7	-0.395E 00	-0.190E-08	-0.128E 00	0.687E-11	-0.311E-03	-0.845E-11
8	0.401E 00	0.165E-09	0.108E 00	0.164E-12	0.311E-03	-0.107E-10
9	0.375E 00	0.292E-08	0.188E 00	0.164E-12	0.311E-03	-0.107E-10
10	0.383E 00	0.200E-08	0.161E 00	0.164E-12	0.311E-03	-0.107E-10
11	0.392E 00	0.108E-08	0.135E 00	0.164E-12	0.311E-03	-0.107E-10
12	0.401E 00	0.165E-09	0.108E 00	0.164E-12	0.311E-03	-0.107E-10
13	-0.412E 00	-0.374E-08	-0.747E-01	0.687E-11	-0.311E-03	-0.845E-11
14	-0.403E 00	-0.282E-08	-0.101E 00	0.687E-11	-0.311E-03	-0.845E-11
15	-0.395E 00	-0.190E-08	-0.128E 00	0.687E-11	-0.311E-03	-0.845E-11
16	0.401E 00	0.165E-09	0.108E 00	0.164E-12	0.311E-03	-0.107E-10
17	0.375E 00	0.292E-08	0.188E 00	0.164E-12	0.311E-03	-0.107E-10
18	0.383E 00	0.200E-08	0.161E 00	0.164E-12	0.311E-03	-0.107E-10
19	0.392E 00	0.108E-08	0.135E 00	0.164E-12	0.311E-03	-0.107E-10
20	0.401E 00	0.165E-09	0.108E 00	0.164E-12	0.311E-03	-0.107E-10
21	-0.412E 00	-0.374E-08	-0.747E-01	0.687E-11	-0.311E-03	-0.845E-11
22	-0.403E 00	-0.282E-08	-0.101E 00	0.687E-11	-0.311E-03	-0.845E-11
23	-0.395E 00	-0.190E-08	-0.128E 00	0.687E-11	-0.311E-03	-0.845E-11
24	0.401E 00	0.165E-09	0.108E 00	0.164E-12	0.311E-03	-0.107E-10
25	0.375E 00	0.292E-08	0.188E 00	0.164E-12	0.311E-03	-0.107E-10
26	0.383E 00	0.200E-08	0.161E 00	0.164E-12	0.311E-03	-0.107E-10
27	0.392E 00	0.108E-08	0.135E 00	0.164E-12	0.311E-03	-0.107E-10
28	0.401E 00	0.165E-09	0.108E 00	0.164E-12	0.311E-03	-0.107E-10
29	-0.412E 00	-0.374E-08	-0.747E-01	0.687E-11	-0.311E-03	-0.845E-11
30	-0.403E 00	-0.282E-08	-0.101E 00	0.687E-11	-0.311E-03	-0.845E-11
31	-0.395E 00	-0.190E-08	-0.128E 00	0.687E-11	-0.311E-03	-0.845E-11
32	0.401E 00	0.165E-09	0.108E 00	0.164E-12	0.311E-03	-0.107E-10
33	0.375E 00	0.292E-08	0.188E 00	0.164E-12	0.311E-03	-0.107E-10
34	0.383E 00	0.200E-08	0.161E 00	0.164E-12	0.311E-03	-0.107E-10
35	0.392E 00	0.108E-08	0.135E 00	0.164E-12	0.311E-03	-0.107E-10
36	0.401E 00	0.165E-09	0.108E 00	0.164E-12	0.311E-03	-0.107E-10
37	-0.412E 00	-0.374E-08	-0.747E-01	0.687E-11	-0.311E-03	-0.845E-11
38	-0.403E 00	-0.282E-08	-0.101E 00	0.687E-11	-0.311E-03	-0.845E-11
39	-0.395E 00	-0.190E-08	-0.128E 00	0.687E-11	-0.311E-03	-0.845E-11
40	0.401E 00	0.165E-09	0.108E 00	0.164E-12	0.311E-03	-0.107E-10
41	0.375E 00	0.292E-08	0.188E 00	0.164E-12	0.311E-03	-0.107E-10
42	0.383E 00	0.200E-08	0.161E 00	0.164E-12	0.311E-03	-0.107E-10
43	0.392E 00	0.108E-08	0.135E 00	0.164E-12	0.311E-03	-0.107E-10
44	0.401E 00	0.165E-09	0.108E 00	0.164E-12	0.311E-03	-0.107E-10
45	-0.412E 00	-0.374E-08	-0.747E-01	0.687E-11	-0.311E-03	-0.845E-11
46	-0.403E 00	-0.282E-08	-0.101E 00	0.687E-11	-0.311E-03	-0.845E-11
47	-0.395E 00	-0.190E-08	-0.128E 00	0.687E-11	-0.311E-03	-0.845E-11
48	0.401E 00	0.165E-09	0.108E 00	0.164E-12	0.311E-03	-0.107E-10
49	0.375E 00	0.292E-08	0.188E 00	0.164E-12	0.311E-03	-0.107E-10
50	0.383E 00	0.200E-08	0.161E 00	0.164E-12	0.311E-03	-0.107E-10
51	0.392E 00	0.108E-08	0.135E 00	0.164E-12	0.311E-03	-0.107E-10
52	0.401E 00	0.165E-09	0.108E 00	0.164E-12	0.311E-03	-0.107E-10
53	-0.412E 00	-0.374E-08	-0.747E-01	0.687E-11	-0.311E-03	-0.845E-11
54	-0.403E 00	-0.282E-08	-0.101E 00	0.687E-11	-0.311E-03	-0.845E-11
55	-0.395E 00	-0.190E-08	-0.128E 00	0.687E-11	-0.311E-03	-0.845E-11
56	0.401E 00	0.165E-09	0.108E 00	0.164E-12	0.311E-03	-0.107E-10
57	0.375E 00	0.292E-08	0.188E 00	0.164E-12	0.311E-03	-0.107E-10
58	0.383E 00	0.200E-08	0.161E 00	0.164E-12	0.311E-03	-0.107E-10
59	0.392E 00	0.108E-08	0.135E 00	0.164E-12	0.311E-03	-0.107E-10
60	0.401E 00	0.165E-09	0.108E 00	0.164E-12	0.311E-03	-0.107E-10

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(Continued)

Table 24 (Continued)

7. PILE FORCES ALONG PILE AXIS (KIPS - FEET)

PILE	F1	F2	F3	F4	F5	F6	CBFTR	FAILURE CB BU CO TE
1	2.843	0.000	100.027	0.	0.	0.	0.37	
2	2.910	0.000	85.886	0.	0.	0.	0.32	
3	2.977	0.000	71.744	0.	0.	0.	0.27	
4	3.045	0.000	57.602	0.	0.	0.	0.21	
5	-3.129	-0.000	-39.859	0.	0.	0.	0.15	
6	-3.062	-0.000	-54.001	0.	0.	0.	0.20	
7	-2.995	-0.000	-68.143	0.	0.	0.	0.25	
8	3.045	0.000	57.602	0.	0.	0.	0.21	
9	2.843	0.000	100.027	0.	0.	0.	0.37	
10	2.910	0.000	85.886	0.	0.	0.	0.32	
11	2.977	0.000	71.744	0.	0.	0.	0.27	
12	3.045	0.000	57.602	0.	0.	0.	0.21	
13	-3.129	-0.000	-39.859	0.	0.	0.	0.15	
14	-3.062	-0.000	-54.001	0.	0.	0.	0.20	
15	-2.995	-0.000	-68.143	0.	0.	0.	0.25	
16	3.045	0.000	57.602	0.	0.	0.	0.21	
17	2.843	0.000	100.027	0.	0.	0.	0.37	
18	2.910	0.000	85.886	0.	0.	0.	0.32	
19	2.977	0.000	71.744	0.	0.	0.	0.27	
20	3.045	0.000	57.602	0.	0.	0.	0.21	
21	-3.129	-0.000	-39.859	0.	0.	0.	0.15	
22	-3.062	-0.000	-54.001	0.	0.	0.	0.20	
23	-2.995	-0.000	-68.143	0.	0.	0.	0.25	
24	3.045	0.000	57.602	0.	0.	0.	0.21	
25	2.843	0.000	100.027	0.	0.	0.	0.37	
26	2.910	0.000	85.886	0.	0.	0.	0.32	
27	2.977	0.000	71.744	0.	0.	0.	0.27	
28	3.045	0.000	57.602	0.	0.	0.	0.21	
29	-3.129	-0.000	-39.859	0.	0.	0.	0.15	
30	-3.062	-0.000	-54.001	0.	0.	0.	0.20	
31	-2.995	-0.000	-68.143	0.	0.	0.	0.25	
32	3.045	0.000	57.602	0.	0.	0.	0.21	
33	2.843	0.000	100.027	0.	0.	0.	0.37	
34	2.910	0.000	85.886	0.	0.	0.	0.32	
35	2.977	0.000	71.744	0.	0.	0.	0.27	
36	3.045	0.000	57.602	0.	0.	0.	0.21	
37	-3.129	-0.000	-39.859	0.	0.	0.	0.15	
38	-3.062	-0.000	-54.001	0.	0.	0.	0.20	
39	-2.995	-0.000	-68.143	0.	0.	0.	0.25	
40	3.045	0.000	57.602	0.	0.	0.	0.21	
41	2.843	0.000	100.027	0.	0.	0.	0.37	
42	2.910	0.000	85.886	0.	0.	0.	0.32	
43	2.977	0.000	71.744	0.	0.	0.	0.27	
44	3.045	0.000	57.602	0.	0.	0.	0.21	
45	-3.129	-0.000	-39.859	0.	0.	0.	0.15	
46	-3.062	-0.000	-54.001	0.	0.	0.	0.20	
47	-2.995	-0.000	-68.143	0.	0.	0.	0.25	
48	3.045	0.000	57.602	0.	0.	0.	0.21	
49	2.843	0.000	100.027	0.	0.	0.	0.37	
50	2.910	0.000	85.886	0.	0.	0.	0.32	
51	2.977	0.000	71.744	0.	0.	0.	0.27	
52	3.045	0.000	57.602	0.	0.	0.	0.21	
53	-3.129	-0.000	-39.859	0.	0.	0.	0.15	
54	-3.062	-0.000	-54.001	0.	0.	0.	0.20	
55	-2.995	-0.000	-68.143	0.	0.	0.	0.25	
56	3.045	0.000	57.602	0.	0.	0.	0.21	
57	2.843	0.000	100.027	0.	0.	0.	0.37	
58	2.910	0.000	85.886	0.	0.	0.	0.32	
59	2.977	0.000	71.744	0.	0.	0.	0.27	
60	3.045	0.000	57.602	0.	0.	0.	0.21	

TOTAL NO. FAILURES = 0   LOAD CASE [illegible]

(Continued)

(Sheet 7 of 11)

Table 24 (Continued)

8. PILE FORCES ALONG STRUCTURE AXIS (KIPS & FEET)

PILE	F1	F2	F3	F4	F5	F6
1	34.329	0.000	93.995	0.	0.	0.
2	29.920	0.000	80.558	0.	0.	0.
3	25.512	0.000	67.121	0.	0.	0.
4	21.104	0.000	53.683	0.	0.	0.
5	15.573	-0.000	-36.824	0.	0.	0.
6	19.981	-0.000	-50.261	0.	0.	0.
7	24.389	-0.000	-63.699	0.	0.	0.
8	21.104	0.000	53.683	0.	0.	0.
9	34.329	0.000	93.995	0.	0.	0.
10	29.920	0.000	80.558	0.	0.	0.
11	25.512	0.000	67.121	0.	0.	0.
12	21.104	0.000	53.683	0.	0.	0.
13	15.573	-0.000	-36.824	0.	0.	0.
14	19.981	-0.000	-50.261	0.	0.	0.
15	24.389	-0.000	-63.699	0.	0.	0.
16	21.104	0.000	53.683	0.	0.	0.
17	34.329	0.000	93.995	0.	0.	0.
18	29.920	0.000	80.558	0.	0.	0.
19	25.512	0.000	67.121	0.	0.	0.
20	21.104	0.000	53.683	0.	0.	0.
21	15.573	-0.000	-36.824	0.	0.	0.
22	19.981	-0.000	-50.261	0.	0.	0.
23	24.389	-0.000	-63.699	0.	0.	0.
24	21.104	0.000	53.683	0.	0.	0.
25	34.329	0.000	93.995	0.	0.	0.
26	29.920	0.000	80.558	0.	0.	0.
27	25.512	0.000	67.121	0.	0.	0.
28	21.104	0.000	53.683	0.	0.	0.
29	15.573	-0.000	-36.824	0.	0.	0.
30	19.981	-0.000	-50.261	0.	0.	0.
31	24.389	-0.000	-63.699	0.	0.	0.
32	21.104	0.000	53.683	0.	0.	0.
33	34.329	0.000	93.995	0.	0.	0.
34	29.920	0.000	80.558	0.	0.	0.
35	25.512	0.000	67.121	0.	0.	0.
36	21.104	0.000	53.683	0.	0.	0.
37	15.573	-0.000	-36.824	0.	0.	0.
38	19.981	-0.000	-50.261	0.	0.	0.
39	24.389	-0.000	-63.699	0.	0.	0.
40	21.104	0.000	53.683	0.	0.	0.
41	34.329	0.000	93.995	0.	0.	0.
42	29.920	0.000	80.558	0.	0.	0.
43	25.512	0.000	67.121	0.	0.	0.
44	21.104	0.000	53.683	0.	0.	0.
45	15.573	-0.000	-36.824	0.	0.	0.
46	19.981	-0.000	-50.261	0.	0.	0.
47	24.389	-0.000	-63.699	0.	0.	0.
48	21.104	0.000	53.683	0.	0.	0.
49	34.329	0.000	93.995	0.	0.	0.
50	29.920	0.000	80.558	0.	0.	0.
51	25.512	0.000	67.121	0.	0.	0.
52	21.104	0.000	53.683	0.	0.	0.
53	15.573	-0.000	-36.824	0.	0.	0.
54	19.981	-0.000	-50.261	0.	0.	0.
55	24.389	-0.000	-63.699	0.	0.	0.
56	21.104	0.000	53.683	0.	0.	0.
57	34.329	0.000	93.995	0.	0.	0.
58	29.920	0.000	80.558	0.	0.	0.
59	25.512	0.000	67.121	0.	0.	0.
60	21.104	0.000	53.683	0.	0.	0.
SUM	1454.250	-0.000	1683.150	-0.000	-2743.534	-0.000

(Continued)

(Sheet 8 of 11)

Table 24 (Continued)

******** LOADING CONDITION 3 ********

****

4. MATRIX OF APPLIED LOADS Q (KIPS & FEET)

Q1	Q2	Q3	Q4	Q5	Q6
1825.950	0.	875.500	0.	-5779.620	0.

****

5. STRUCTURE DEFLECTIONS (INCHES)

D1	D2	D3	D4	D5	D6
0.552E 00	0.279E-08	-0.239E-01	-0.367E-11	0.325E-03	-0.119E-10

****

6. PILE DEFLECTIONS ALONG PILE AXIS (INCHES)

PILE	X1	X2	X3	X4	X5	X6
1	0.517E 00	0.440E-08	0.193E 00	0.290E-12	0.325E-03	-0.125E-10
2	0.526E 00	0.332E-08	0.166E 00	0.290E-12	0.325E-03	-0.125E-10
3	0.535E 00	0.225E-08	0.138E 00	0.290E-12	0.325E-03	-0.125E-10
4	0.545E 00	0.118E-08	0.110E 00	0.290E-12	0.325E-03	-0.125E-10
5	-0.530E 00	-0.549E-08	-0.155E 00	0.787E-11	-0.325E-03	-0.996E-11
6	-0.520E 00	-0.442E-08	-0.183E 00	0.787E-11	-0.325E-03	-0.996E-11
7	-0.511E 00	-0.334E-08	-0.211E 00	0.787E-11	-0.325E-03	-0.996E-11
8	0.545E 00	0.118E-08	0.110E 00	0.290E-12	0.325E-03	-0.125E-10
9	0.517E 00	0.440E-08	0.193E 00	0.290E-12	0.325E-03	-0.125E-10
10	0.526E 00	0.332E-08	0.166E 00	0.290E-12	0.325E-03	-0.125E-10
11	0.535E 00	0.225E-08	0.138E 00	0.290E-12	0.325E-03	-0.125E-10
12	0.545E 00	0.118E-08	0.110E 00	0.290E-12	0.325E-03	-0.125E-10
13	-0.530E 00	-0.549E-08	-0.155E 00	0.787E-11	-0.325E-03	-0.996E-11
14	-0.520E 00	-0.442E-08	-0.183E 00	0.787E-11	-0.325E-03	-0.996E-11
15	-0.511E 00	-0.334E-08	-0.211E 00	0.787E-11	-0.325E-03	-0.996E-11
16	0.545E 00	0.118E-08	0.110E 00	0.290E-12	0.325E-03	-0.125E-10
17	0.517E 00	0.440E-08	0.193E 00	0.290E-12	0.325E-03	-0.125E-10
18	0.526E 00	0.332E-08	0.166E 00	0.290E-12	0.325E-03	-0.125E-10
19	0.535E 00	0.225E-08	0.138E 00	0.290E-12	0.325E-03	-0.125E-10
20	0.545E 00	0.118E-08	0.110E 00	0.290E-12	0.325E-03	-0.125E-10
21	-0.530E 00	-0.549E-08	-0.155E 00	0.787E-11	-0.325E-03	-0.996E-11
22	-0.520E 00	-0.442E-08	-0.183E 00	0.787E-11	-0.325E-03	-0.996E-11
23	-0.511E 00	-0.334E-08	-0.211E 00	0.787E-11	-0.325E-03	-0.996E-11
24	0.545E 00	0.118E-08	0.110E 00	0.290E-12	0.325E-03	-0.125E-10
25	0.517E 00	0.440E-08	0.193E 00	0.290E-12	0.325E-03	-0.125E-10
26	0.526E 00	0.332E-08	0.166E 00	0.290E-12	0.325E-03	-0.125E-10
27	0.535E 00	0.225E-08	0.138E 00	0.290E-12	0.325E-03	-0.125E-10
28	0.545E 00	0.118E-08	0.110E 00	0.290E-12	0.325E-03	-0.125E-10
29	-0.530E 00	-0.549E-08	-0.155E 00	0.787E-11	-0.325E-03	-0.996E-11
30	-0.520E 00	-0.442E-08	-0.183E 00	0.787E-11	-0.325E-03	-0.996E-11
31	-0.511E 00	-0.334E-08	-0.211E 00	0.787E-11	-0.325E-03	-0.996E-11
32	0.545E 00	0.118E-08	0.110E 00	0.290E-12	0.325E-03	-0.125E-10
33	0.517E 00	0.440E-08	0.193E 00	0.290E-12	0.325E-03	-0.125E-10
34	0.526E 00	0.332E-08	0.166E 00	0.290E-12	0.325E-03	-0.125E-10
35	0.535E 00	0.225E-08	0.138E 00	0.290E-12	0.325E-03	-0.125E-10
36	0.545E 00	0.118E-08	0.110E 00	0.290E-12	0.325E-03	-0.125E-10
37	-0.530E 00	-0.549E-08	-0.155E 00	0.787E-11	-0.325E-03	-0.996E-11
38	-0.520E 00	-0.442E-08	-0.183E 00	0.787E-11	-0.325E-03	-0.996E-11
39	-0.511E 00	-0.334E-08	-0.211E 00	0.787E-11	-0.325E-03	-0.996E-11
40	0.545E 00	0.118E-08	0.110E 00	0.290E-12	0.325E-03	-0.125E-10
41	0.517E 00	0.440E-08	0.193E 00	0.290E-12	0.325E-03	-0.125E-10
42	0.526E 00	0.332E-08	0.166E 00	0.290E-12	0.325E-03	-0.125E-10
43	0.535E 00	0.225E-08	0.138E 00	0.290E-12	0.325E-03	-0.125E-10
44	0.545E 00	0.118E-08	0.110E 00	0.290E-12	0.325E-03	-0.125E-10
45	-0.530E 00	-0.549E-08	-0.155E 00	0.787E-11	-0.325E-03	-0.996E-11
46	-0.520E 00	-0.442E-08	-0.183E 00	0.787E-11	-0.325E-03	-0.996E-11
47	-0.511E 00	-0.334E-08	-0.211E 00	0.787E-11	-0.325E-03	-0.996E-11
48	0.545E 00	0.118E-08	0.110E 00	0.290E-12	0.325E-03	-0.125E-10
49	0.517E 00	0.440E-08	0.193E 00	0.290E-12	0.325E-03	-0.125E-10
50	0.526E 00	0.332E-08	0.166E 00	0.290E-12	0.325E-03	-0.125E-10
51	0.535E 00	0.225E-08	0.138E 00	0.290E-12	0.325E-03	-0.125E-10
52	0.545E 00	0.118E-08	0.110E 00	0.290E-12	0.325E-03	-0.125E-10
53	-0.530E 00	-0.549E-08	-0.155E 00	0.787E-11	-0.325E-03	-0.996E-11
54	-0.520E 00	-0.442E-08	-0.183E 00	0.787E-11	-0.325E-03	-0.996E-11
55	-0.511E 00	-0.334E-08	-0.211E 00	0.787E-11	-0.325E-03	-0.996E-11
56	0.545E 00	0.118E-08	0.110E 00	0.290E-12	0.325E-03	-0.125E-10
57	0.517E 00	0.440E-08	0.193E 00	0.290E-12	0.325E-03	-0.125E-10
58	0.526E 00	0.332E-08	0.166E 00	0.290E-12	0.325E-03	-0.125E-10
59	0.535E 00	0.225E-08	0.138E 00	0.290E-12	0.325E-03	-0.125E-10
60	0.545E 00	0.118E-08	0.110E 00	0.290E-12	0.325E-03	-0.125E-10

****

(Continued)

(Sheet 9 of 11)

Table 24 (Continued)

7. PILE FORCES ALONG PILE AXIS (KIPS & FEET)

PILE	F1	F2	F3	F4	F5	F6	CSFTR	FAILURE CB	BU	CO	TE
1	3.923	0.000	103.139	0.	0.	0.	0.38				
2	3.993	0.000	88.319	0.	0.	0.	0.33				
3	4.064	0.000	75.499	0.	0.	0.	0.27				
4	4.134	0.000	58.079	0.	0.	0.	0.22				
5	-4.019	-0.000	-82.894	0.	0.	0.	0.31				
6	-3.949	-0.000	-97.714	0.	0.	0.	0.36				
7	-3.879	-0.000	-112.534	0.	0.	0.	0.42				F
8	4.134	0.000	58.679	0.	0.	0.	0.22				
9	3.923	0.000	103.139	0.	0.	0.	0.38				
10	3.993	0.000	88.319	0.	0.	0.	0.33				
11	4.064	0.000	73.499	0.	0.	0.	0.27				
12	4.134	0.000	58.079	0.	0.	0.	0.22				
13	-4.019	-0.000	-82.894	0.	0.	0.	0.31				
14	-3.949	-0.000	-97.714	0.	0.	0.	0.36				
15	-3.879	-0.000	-112.534	0.	0.	0.	0.42				F
16	4.134	0.000	58.079	0.	0.	0.	0.22				
17	3.923	0.000	103.139	0.	0.	0.	0.38				
18	3.993	0.000	88.319	0.	0.	0.	0.33				
19	4.064	0.000	73.499	0.	0.	0.	0.27				
20	4.134	0.000	58.079	0.	0.	0.	0.22				
21	-4.019	-0.000	-82.894	0.	0.	0.	0.31				
22	-3.949	-0.000	-97.714	0.	0.	0.	0.36				
23	-3.879	-0.000	-112.534	0.	0.	0.	0.42				F
24	4.134	0.000	58.079	0.	0.	0.	0.22				
25	3.923	0.000	103.139	0.	0.	0.	0.38				
26	3.993	0.000	88.319	0.	0.	0.	0.33				
27	4.064	0.000	73.499	0.	0.	0.	0.27				
28	4.134	0.000	58.679	0.	0.	0.	0.22				
29	-4.019	-0.000	-82.894	0.	0.	0.	0.31				
30	-3.949	-0.000	-97.714	0.	0	0.	0.36				
31	-3.879	-0.000	-112.534	0.	0.	0.	0.42				F
32	4.134	0.000	58.079	0.	0.	0.	0.22				
33	3.923	0.000	103.139	0.	0.	0.	0.38				
34	3.993	0.000	88.319	0.	0.	0.	0.33				
35	4.064	0.000	73.499	0.	0.	0.	0.27				
36	4.134	0.000	58.079	0.	0.	0.	0.22				
37	-4.019	-0.000	-82.894	0.	0.	0.	0.31				
38	-3.949	-0.000	-97.714	0.	0.	0.	0.36				
39	-3.879	-0.000	-112.534	0.	0.	0.	0.42				F
40	4.134	0.000	58.679	0.	0.	0.	0.22				
41	3.923	0.000	103.139	0.	0.	0.	0.38				
42	3.993	0.000	88.319	0.	0.	0.	0.33				
43	4.064	0.000	73.499	0.	0.	0.	0.27				
44	4.134	0.000	58.079	0.	0.	0.	0.22				
45	-4.019	-0.000	-82.894	0.	0.	0.	0.31				
46	-3.949	-0.000	-97.714	0.	0.	0.	0.36				
47	-3.879	-0.000	-112.534	0.	0.	0.	0.42				F
48	4.134	0.000	58.079	0.	0.	0.	0.22				
49	3.923	0.000	103.139	0.	0.	0.	0.38				
50	3.993	0.000	88.319	0.	0.	0.	0.33				
51	4.064	0.000	73.499	0.	0.	0.	0.27				
52	4.134	0.000	58.079	0.	0.	0.	0.22				
53	-4.019	-0.000	-82.894	0.	0.	0.	0.31				
54	-3.949	-0.000	-97.714	0.	0.	0.	0.36				
55	-3.879	-0.000	-112.534	0.	0.	0.	0.42				F
56	4.134	0.000	58.079	0.	0.	0.	0.22				
57	3.923	0.000	103.139	0.	0.	0.	0.38				
58	3.993	0.000	88.319	0.	0.	0.	0.33				
59	4.064	0.000	73.499	0.	0.	0.	0.27				
60	4.134	0.000	58.079	0.	0.	0.	0.22				

TOTAL NO. FAILURES = 7 LOAD CASE [illegible]

****

(Continued)

(Sheet 10 of 11)

Table 24 (Concluded)

8. PILE FORCES ALONG STRUCTURE AXIS (KIPS & FEET)

PILE	F1	F2	F3	F4	F5	F6
1	36.337	0.000	96.605	0.	0.	0.
2	31.717	0.000	82.524	0.	0.	0.
3	27.098	0.000	68.442	0.	0.	0.
4	22.478	0.000	54.360	0.	0.	0.
5	30.026	-0.000	-77.370	0.	0.	0.
6	34.646	-0.000	-91.451	0.	0.	0.
7	39.266	-0.000	-105.533	0.	0.	0.
8	22.478	0.000	54.360	0.	0.	0.
9	36.337	0.000	96.605	0.	0.	0.
10	31.717	0.000	82.524	0.	0.	0.
11	27.098	0.000	68.442	0.	0.	0.
12	22.478	0.000	54.360	0.	0.	0.
13	30.026	-0.000	-77.370	0.	0.	0.
14	34.646	-0.000	-91.451	0.	0.	0.
15	39.266	-0.000	-105.533	0.	0.	0.
16	22.478	0.000	54.360	0.	0.	0.
17	36.337	0.000	96.605	0.	0.	0.
18	31.717	0.000	82.524	0.	0.	0.
19	27.098	0.000	68.442	0.	0.	0.
20	22.478	0.000	54.360	0.	0.	0.
21	30.026	-0.000	-77.370	0.	0.	0.
22	34.646	-0.000	-91.451	0.	0.	0.
23	39.266	-0.000	-105.533	0.	0.	0.
24	22.478	0.000	54.360	0.	0.	0.
25	36.337	0.000	96.605	0.	0.	0.
26	31.717	0.000	82.524	0.	0.	0.
27	27.098	0.000	68.442	0.	0.	0.
28	22.478	0.000	54.360	0.	0.	0.
29	30.026	-0.000	-77.370	0.	0.	0.
30	34.646	-0.000	-91.451	0.	0.	0.
31	39.266	-0.000	-105.533	0.	0.	0.
32	22.478	0.000	54.360	0.	0.	0.
33	36.337	0.000	96.605	0.	0.	0.
34	31.717	0.000	82.524	0.	0.	0.
35	27.098	0.000	68.442	0.	0.	0.
36	22.478	0.000	54.360	0.	0.	0.
37	30.026	-0.000	-77.370	0.	0.	0.
38	34.646	-0.000	-91.451	0.	0.	0.
39	39.266	-0.000	-105.533	0.	0.	0.
40	22.478	0.000	54.360	0.	0.	0.
41	36.337	0.000	96.605	0.	0.	0.
42	31.717	0.000	82.524	0.	0.	0.
43	27.098	0.000	68.442	0.	0.	0.
44	22.478	0.000	54.360	0.	0.	0.
45	30.026	-0.000	-77.370	0.	0.	0.
46	34.646	-0.000	-91.451	0.	0.	0.
47	39.266	-0.000	-105.533	0.	0.	0.
48	22.478	0.000	54.360	0.	0.	0.
49	36.337	0.000	96.605	0.	0.	0.
50	31.717	0.000	82.524	0.	0.	0.
51	27.098	0.000	68.442	0.	0.	0.
52	22.478	0.000	54.360	0.	0.	0.
53	30.026	-0.000	-77.370	0.	0.	0.
54	34.646	-0.000	-91.451	0.	0.	0.
55	39.266	-0.000	-105.533	0.	0.	0.
56	22.478	0.000	54.360	0.	0.	0.
57	36.337	0.000	96.605	0.	0.	0.
58	31.717	0.000	82.524	0.	0.	0.
59	27.098	0.000	68.442	0.	0.	0.
60	22.478	0.000	54.360	0.	0.	0.
SUM	1825.950	-0.000	875.500	-0.000	-5779.620	-0.001

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(Sheet 11 of 11)

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APPENDIX A: USER'S GUIDE FOR PROGRAM PILESTF

General Introduction

1. Documentation for the computer program PILESTF--to compute the pile head stiffness matrix for a linearly elastic pile-soil system--is presented in this appendix and includes a general introduction, previous work, general pile head stiffness analysis, guide for data input, and input-output data for example problems.

2. PILESTF is a finite element computer code (developed by Dr. William P. Dawkins) that can solve for the pile head stiffness matrix. The procedure used is a one-dimensional finite element analysis of a beam on an elastic foundation. The pile is replaced by a linearly elastic system of springs (pile stiffness coefficients) which describe the resistance of the pile to displacements of the structure.

3. PILESTF can be run on the WES G-635, Macon H-6000, and Boeing CDC computers in the time-sharing mode. The program is part of the CORPS library and is identified by the program number X0035. To execute the program, issue one of the following appropriate run commands. On the WES or Macon computer,

```
RUN WESLIB/CORPS/X0035,R
```

On the Boeing computer,

```
OLD,CORPS/UN=CECELB
CALL, CORPS, X0035
```

Data must be input interactively at execute time. Output comes directly back to the terminal.

Background

4. A typically laterally loaded pile and the notation used in this report are shown in Figure A1. Only a 2D system is shown; extension to three dimensions in immediate. The relationship between forces applied to the pile head and the resulting displacements may be expressed as

$$\begin{Bmatrix} F_x \\ F_z \\ M_y \end{Bmatrix} = \begin{bmatrix} b_{11} & 0 & b_{13} \\ 0 & b_{22} & 0 \\ b_{31} & 0 & b_{33} \end{bmatrix} \begin{Bmatrix} u \\ w \\ \theta \end{Bmatrix} \qquad \text{(A1)}$$

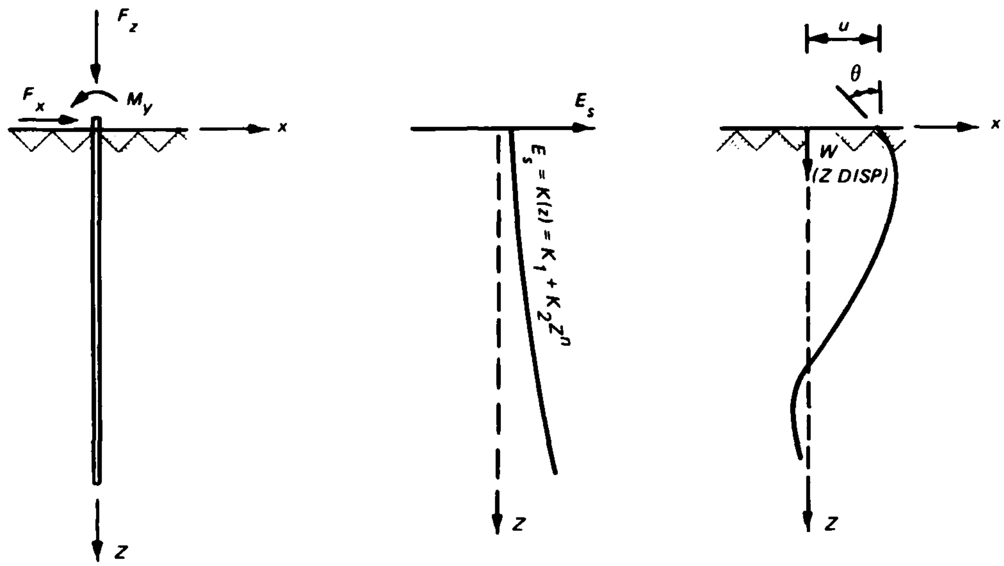

Figure A1. Notation for laterally loaded piles

The [b] matrix in Equation A1 is the pile head stiffness matrix and each term in the matrix, b_{ij}, is equal to the force of type i produced by a unit displacement of type j with all other displacements equal to zero. Because the pile-soil system is assumed to be linearly elastic, energy must be conserved which requires that $b_{ij} = b_{ji}$ for every i and j.

5. Assessment of the values of the b_{ij} coefficients has been based on the finite difference solutions by Reese and Matlock (1956) of the fourth-order differential equation

$$EI \frac{d^4u}{dz^4} + k(z)\ u = 0 \tag{A2}$$

where

EI = bending rigidity of the pile

u = lateral displacement

z = distance along the pile

k(z) = soil modulus which may be a function of z as shown in Figure A1.

6. Reese and Matlock (1956) have expressed the lateral force-deformation behavior in the form

$$u = \frac{F_x T^3}{EI} A_u - \frac{M_y T^2}{EI} B_u \tag{A3}$$

and

$$\theta = -\frac{F_x T^2}{EI} A_\theta + \frac{M_y T^2}{EI} B_\theta \tag{A4}$$

where, for instance, with $k(z) = k_2 z^n$,

$$T = \sqrt[4+n]{\frac{EI}{k_2}}$$

and A_u , B_u , A_θ , B_θ are coefficients which depend on the relative magnitudes of EI , $k(z)$, and pile length. Charts giving values of A_u , B_u , A_θ , B_θ for a variety of soil-pile parameters are given in Reese and Matlock (1956).

7. The stiffness terms b_{ij} are readily obtained from Equations A3 and A4 and the tabulated coefficients of Reese and Matlock (1956) by imposing successive unit values of u and θ and evaluating the resulting forces F_x and M_y .

8. Determination of the b_{ij} for extreme cases of a rigid pile-structure connection or a pinned pile-structure connection is straightforward. However, an anomaly arises when the pile-structure connection is assumed to be only partially effective in resisting moment. A previously used method and an alternative procedure for evaluating the b_{ij} stiffness terms for partially fixed head piles are described below.

Previous Pile Head Stiffness Evaluation

9. Niemi (1976) presents a procedure for determining the b_{ij} coefficients under the assumption that the pile is "infinitely" long and the pile-structure connection is capable of producing only a

fraction (DF) of the resisting moment of a completely fixed head pile. First, the relationship between the moment developed in a completely fixed pile and a unit value of lateral displacement ($u = 1$, $\theta = 0$) is determined.

10. From Equation A4 for $\theta = 0$,

$$M_{yf} = \frac{A_\theta}{B_\theta} F_x T \tag{A5}$$

The assumption is then made that the pile head produces for any displacement u,

$$M_y = (DF)\, M_{yf} \tag{A6}$$

where $0 \leq (DF) \leq 1$.

11. From Equations A3 and A6,

$$u = \frac{F_x T^3}{EI} \left[A_u - (DF) \frac{A_\theta}{B_\theta} B_u \right] \tag{A7}$$

12. By definition,

$$b_{11} = \frac{F_x}{u} = K_1 \frac{EI}{T^3}$$

where

$$K_1 = \frac{1}{A_u - (DF) \dfrac{A_\theta}{B_\theta} B_u} \tag{A8}$$

13. Similarly from Equations A5, A6, and A7, by definition,

$$b_{31} = \frac{M_y}{u} = \frac{(DF) M_{yf}}{u}$$

or

$$b_{31} = K_2 \frac{EI}{T^2}$$

where

$$K_2 = \frac{(DF)}{\frac{B_\theta}{A_\theta} A_u - (DF)\ B_u} \qquad (A9)$$

14. For evaluation of coefficients b_{13} and b_{33} , the assumption is made that the pile head moment for $\theta = 1$, $u = 0$ is

$$M_y = (DF)\ M_{yf} \qquad (A10)$$

15. From Equation A3 with $u = 0$,

$$F_x = \frac{B_u}{A_u} \frac{M_{yf}}{T} \qquad (A11)$$

And, from Equations A4 and A11,

$$\theta = \frac{M_{yf} T}{EI} \left[B_\theta - \frac{B_u}{A_u} A_\theta \right] \qquad (A12)$$

16. By definition, from Equations A10 and A12,

$$b_{33} = \frac{M_y}{\theta} = \frac{(DF) M_{yf}}{\theta} = K_3 \frac{EI}{T}$$

where

$$K_3 = \frac{(DF)}{B_\theta - \frac{B_u}{A_u} A_\theta} \qquad (A13)$$

17. Also, from Equations A11 and A12, as defined in this procedure,

$$B_{13} = \frac{(DF) F_x}{\theta} = (DF)\ \frac{B_u}{A_u} \frac{M_{yf}}{\theta} = K_4 \frac{EI}{T^2}$$

where

$$K_4 = \frac{DF}{\frac{A_u}{B_u} B_\theta - A_\theta} \qquad (A14)$$

18. In summary the pile head stiffness matrix established by this procedure is

$$[b] = \begin{bmatrix} \left[\dfrac{1}{A_u - (DF)\dfrac{A_\theta}{B_\theta} B_u} \right] \dfrac{EI}{T^3} & 0 & \left[\dfrac{(DF)}{\dfrac{A_u}{B_u} B_\theta - A_\theta} \right] \dfrac{EI}{T^2} \\ 0 & b_{22}{}^{*} & 0 \\ \left[\dfrac{(DF)}{\dfrac{B_\theta}{A_\theta} A_u - (DF)\, B_u} \right] \dfrac{EI}{T^2} & 0 & \left[\dfrac{(DF)}{B_\theta - \dfrac{B_u}{A_u} A_\theta} \right] \dfrac{EI}{T} \end{bmatrix} \qquad \text{(A15)}$$

19. Noting that $A_\theta = B_u$ (see Reese and Matlock (1956)), the term b_{31} may be written as

$$b_{31} = \frac{(DF)}{\dfrac{A_u}{B_u} B_\theta - (DF)\, B_u}$$

20. It is apparent that except for pinned head piles ((DF) = 0) or fixed head piles ((DF) = 1), the pile head stiffness matrix developed by this procedure is unsymmetric and therefore violates the requirement of conservation of energy.

21. It is further to be noted that the effect of partial fixity is different for resistance to lateral translation u that for rotation θ . In the stiffness matrix, Equation A15, resistance to rotation is directly proportional to (DF) while resistance to translation is inversely proportional to (DF).

---

* b_{22} is the axial stiffness of the pile and is determined separately from the lateral force-displacement effects by procedures which are not covered in this appendix.

Alternate Derivation

22. For a pinned head pile, ((DF) = 0), Equations A3 and A4 yield (with $u = 1$, $\theta = \theta_{free}$, $M_y = 0$)

$$1 = \frac{F_x T^3}{EI} A_u$$

and

$$|\theta_{free}| = \frac{F_x T^2}{EI} A_\theta = \frac{A}{A_u} \frac{1}{T} \qquad \text{(A16)}$$

23. For a partially restrained pile it is assumed that moment resistance develops at a reduced rate (proportional to $(DF)\cdot\theta_{free}$). Then from Equations A3 and A4,

$$1 = \frac{F_x T^3}{EI} A_u - \frac{M_y T^2}{EI} B_u \qquad \text{(A17)}$$

$$\theta = -(1 - DF)|\theta_{free}| = -(1 - DF) \frac{A_\theta}{A_u} \frac{1}{T} = - \frac{F_x T^2}{EI} A_\theta + \frac{M_y T}{EI} B_\theta \qquad \text{(A18)}$$

Equations A17 and A18 may be solved simultaneously to find .

$$b_{11} = F_x = K_1 \frac{EI}{T^3}$$

where

$$K_1 = \frac{1}{A_u} \left[1 + \frac{(DF)}{\frac{A_u}{B_u} \frac{B}{A_u} - 1} \right] \qquad \text{(A19)}$$

$$b_{31} = M_y = K_2 \frac{EI}{T^2}$$

where

$$K_2 = \frac{(DF)}{\frac{B_\theta}{A_\theta} A_u - B_u} \tag{A20}$$

24. For b_{13} and b_{33} , displacements $u = 0$ and $\theta = (DF)$ are imposed at the pile head. Then from Equations A3 and A4,

$$0 = \frac{F_x T^3}{EI} A_u - \frac{M_y T^2}{EI} B_u \tag{A21}$$

$$(DF) = - \frac{F_x T^2}{EI} A_\theta + \frac{M_y T}{EI} B_\theta \tag{A22}$$

Equations A21 and A22 may be solved simultaneously to find

$$b_{33} = M_y = K_3 \frac{EI}{T}$$

where

$$K_3 = \frac{(DF)}{B_\theta - \frac{B_u}{A_u} A_\theta} \tag{A23}$$

and

$$b_{13} = F_x = K_4 \frac{EI}{T^2}$$

where

$$K_4 = \frac{(DF)}{\frac{A_u}{B_u} B_\theta - A_\theta} \tag{A24}$$

25. In summary, by the alternate procedure, the following pile head stiffness matrix is obtained:

$$[b] = \begin{bmatrix} \frac{1}{A_u}\left[1 + \frac{(DF)}{\frac{A_u}{B_u}\frac{B_\theta}{A_\theta} - 1}\right]\frac{EI}{T^3} & 0 & \left[\frac{(DF)}{\frac{A_u}{B_u}B_\theta - A_\theta}\right]\frac{EI}{T^2} \\ 0 & b_{22} & 0 \\ \left[\frac{(DF)}{\frac{B_\theta}{A_\theta}A_u - B_u}\right]\frac{EI}{T^2} & 0 & \left[\frac{(DF)}{B_\theta - \frac{B_u}{A_u}A_\theta}\right]\frac{EI}{T} \end{bmatrix} \qquad (A25)$$

Noting that $A_\theta = B_u$, the above stiffness matrix is symmetric and all coefficients are directly proportional to (DF).

26. The values obtained for K_3 and K_4 are the same by either procedure. Values from the two procedures for K_1 and K_2 are compared in Figure A2 for the particular case of an "infinitely" long pile

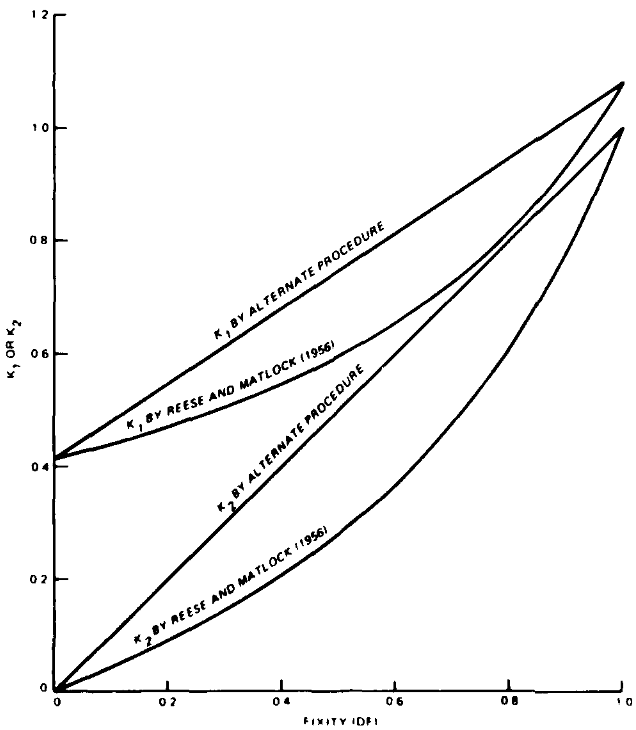

Figure A2. Comparison of stiffness coefficients

with soil modulus $k(z) = k_2 z$. (From Reese and Matlock (1956) and Niemi (1976) for this case, $A_u = 2.435$, $B_u = A_\theta = 1.623$, $B_\theta = 1.749$.)

General Pile Head Stiffness Analysis Introduction

27. The procedures discussed above may be used to develop pile head stiffness matrices provided that appropriate values of A_u , B_u , A_θ , B_θ are available for the particular combinations of EI , $k(z)$, and pile length under consideration. Reese and Matlock (1956) provide these coefficients for only a limited number of variations of pile-soil parameters. In the remainder of this appendix a procedure and attendant computer program are described which permit development of pile head stiffness matrices for either two- or three-dimensional pile-soil systems for any combination of pile-soil parameters. It was anticipated that the pile head stiffness matrices developed by the program would subsequently be used as input for general purpose structural analysis programs. Because many of these general purpose programs do not accommodate unsymmetric stiffness matrices, the alternate procedure for partial fixity described in previous section was adopted.

Pile-Soil Model

28. The procedure used in this appendix is a one-dimensional finite element analysis of a beam on an elastic foundation. The continuous pile-soil system is replaced by a beam resting on discrete springs as shown in Figure A3a and A3b. Freebody diagrams of a general node i and adjacent elements i and i+1 are shown in Figure A3c. (Note: subscripts refer to nodes, superscripts refer to elements; e.g., f_i^{i+1} is the shear force at node i in element i+1 .) Each node undergoes a translation u_i in the x-direction and a rotation θ_i about a y-axis, where x and y are principal axes of the cross section. External nodal forces $F_{x,i}$ and $M_{y,i}$ are assumed to act at each node, although all nodal forces except at the pile head will subsequently be

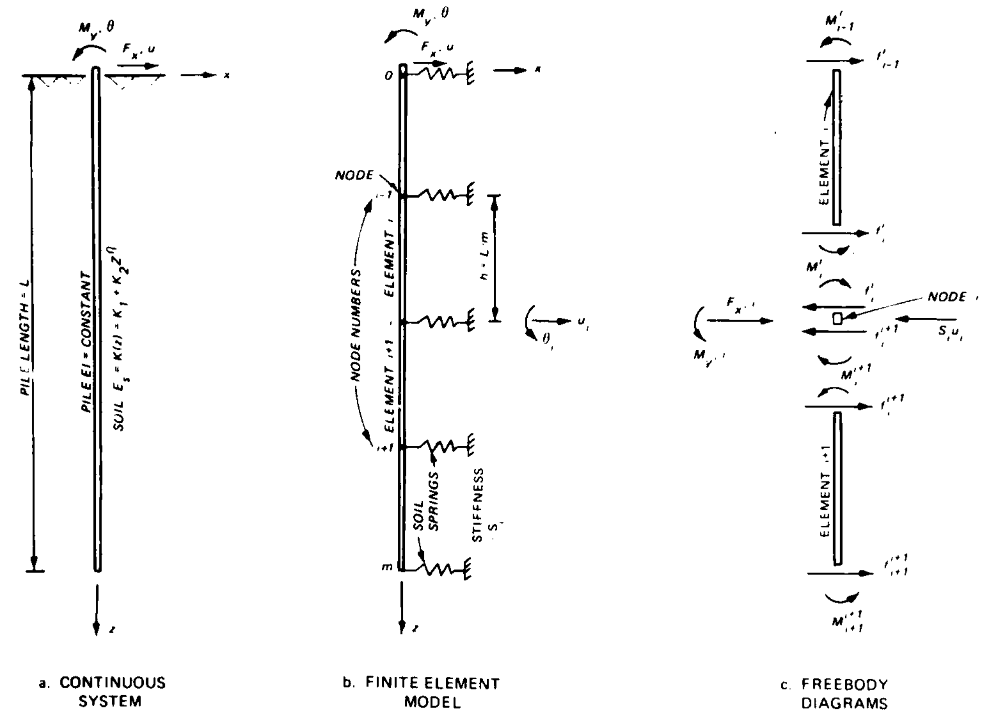

Figure A3. Finite element model of pile-soil system

set to zero. The soil spring at each node produces a force which resists displacement equal to the product of the spring stiffness S_i and the x nodal displacement u_i .

Soil Springs

29. Before analysis of the finite element model can be performed, the stiffness S_i , Figure A3, must be established from the properties of the surrounding soil. At any node the soil modulus is

$$k(z)_i = k_1 + k_2 z^n = k_1 + k_2 (ih)^n$$

A weighted averaging process is used to convert the soil modulus to discrete spring stiffnesses as follows.

At $i = 0$ (pile head):

$$S_o = \frac{h}{6}\left[2k(z)_o + k(z)_1\right]$$

For $1 \leq i \leq m - 1$:

$$S_i = \frac{h}{6}\left[k(z)_{i-1} + 4k(z)_i + k(z)_{i+1}\right]$$

And for $i = m$ (bottom of pile):

$$S_m = \frac{h}{6}\left[k(z)_{m-1} + 2k(z)_m\right] .$$

Element Force-Displacement Relations

30. The end force-displacement relations are obtained from ordinary beam analysis and may be expressed as follows. For element i , at node i:

$$\begin{Bmatrix} f_i^i \\ m_i^i \end{Bmatrix} = \begin{bmatrix} \frac{-12EI}{h^3} & \frac{-6EI}{h^2} \\ \frac{6EI}{h^2} & \frac{2EI}{h} \end{bmatrix} \begin{Bmatrix} u_{i-1} \\ \theta_{i-1} \end{Bmatrix} + \begin{bmatrix} \frac{12EI}{h^3} & \frac{-6EI}{h^2} \\ \frac{-6EI}{h^2} & \frac{4EI}{h} \end{bmatrix} \begin{Bmatrix} u_i \\ \theta_i \end{Bmatrix} \tag{A26}$$

or

$$\underset{\sim}{f}_i^i = \underset{\sim}{a}_i \underset{\sim}{U}_{i-1} + \underset{\sim}{b}_i^i \underset{\sim}{U}_i \tag{A27}$$

And, for element $i+1$, at node i:

$$\begin{Bmatrix} f_i^{i+1} \\ m_i^{i+1} \end{Bmatrix} = \begin{bmatrix} \frac{12EI}{h^3} & \frac{6EI}{h^2} \\ \frac{6EI}{h^2} & \frac{4EI}{h} \end{bmatrix} \begin{Bmatrix} u_i \\ \theta_i \end{Bmatrix} + \begin{bmatrix} \frac{-12EI}{h^3} & \frac{6EI}{h^2} \\ \frac{-6EI}{h^2} & \frac{2EI}{h} \end{bmatrix} \begin{Bmatrix} u_{i+1} \\ \theta_{i+1} \end{Bmatrix} \tag{A28}$$

or

$$\underset{\sim}{f}_i^{i+1} = \underset{\sim}{b}_i^{i+1}\underset{\sim}{U}_i + \underset{\sim}{c}_i\underset{\sim}{U}_{i+1} \tag{A29}$$

Nodal Equilibrium

31. For equilibrium at the ith node:

$$f_i^i + f_i^{i+1} + S_i u_i = F_{x,i} \tag{A30}$$

$$m_i^i + m_i^{i+1} = M_{y,i} \tag{A31}$$

which may be written as

$$\begin{Bmatrix} f_i^i \\ m_i \end{Bmatrix} + \begin{Bmatrix} f_i^{i+1} \\ m_i^{i+1} \end{Bmatrix} + \begin{bmatrix} S_i & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} u_i \\ \theta_i \end{Bmatrix} = \begin{Bmatrix} F_{x,i} \\ M_{y,i} \end{Bmatrix} \tag{A32}$$

Or, using the notation of Equations A27 and A29, and introducing

$$\underset{\sim}{S}_i = \begin{bmatrix} S_i & 0 \\ 0 & 0 \end{bmatrix}$$

Equation A28 may be written as

$$\underset{\sim}{f}_i^i + \underset{\sim}{f}_i^{i+1} + \underset{\sim}{S}_i\underset{\sim}{U}_i = \underset{\sim}{F}_i \tag{A33}$$

Finally, combination of Equations A27, A29, and A32 yields

$$\underset{\sim}{a}_i\underset{\sim}{U}_{i-1} + \left(\underset{\sim}{b}_i^i + \underset{\sim}{b}_i^{i+1} + \underset{\sim}{S}_i\right)\underset{\sim}{U}_i + \underset{\sim}{c}_i\underset{\sim}{U}_{i+1} = \underset{\sim}{F}_i = \underset{\sim}{0} \tag{A34}$$

Equation A34 must be satisfied at every node, $1 \le i \le m - 1$ (note: $F_i = 0$, since no external nodal forces are applied except at the pile head).

Special Conditions at Node m (Bottom of Pile)

32. At node m, because element m+1 does not exist, Equation A34 reduces to

$$\underset{\sim}{a}_m \underset{\sim}{U}_{m-1} + \left(\underset{\sim}{b}_m^m + \underset{\sim}{S}_m\right)\underset{\sim}{U}_m = \underset{\sim}{F}_m = \underset{\sim}{0} \ . \qquad \text{(A35)}$$

Special Conditions at Node o (Pile Head)

33. Because no element exists above node o, Equation A32 reduces to

$$\left(\underset{\sim}{b}_o^1 + \underset{\sim}{S}_o\right)\underset{\sim}{U}_o + \underset{\sim}{c}_o\underset{\sim}{U}_1 = \underset{\sim}{F}_o \ . \qquad \text{(A36)}$$

34. Equations A34, A35, and A36 represent m+1 simultaneous equations which relate pile head forces ($F_{x,o}$ and $M_{y,o}$) to displacements along the pile. In order to develop the pile head stiffness matrix, particular combinations of pile head displacements, u_o and θ_o ($M_{y,o}$ for a pinned head pile), are imposed. The forces, $F_{x,o}$ and $M_{y,o}$, resulting from these specified conditions are, by definition, elements of the desired stiffness matrix.

35. For a pinned head pile, the conditions to be specified at the pile head are $u_o = 1$ and $M_{y,o} = 0$. To reflect these conditions Equation A36 (see also Equation A28) must be altered to

$$\begin{bmatrix} 1 & 0 \\ \frac{-6EI}{h^2} & \frac{2EI}{h} \end{bmatrix} \begin{Bmatrix} u_o \\ \theta_o \end{Bmatrix} + \begin{bmatrix} 0 & 0 \\ \frac{-6EI}{h^2} & \frac{4EI}{h} \end{bmatrix} \begin{Bmatrix} u_1 \\ \theta_1 \end{Bmatrix} = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \qquad \text{(A37)}$$

The displacements obtained from the solution of Equations A34, A35, and A37 are then substituted into Equations A26, A28, and A32 (with i = 0) to obtain

$$F_{x,o} = \frac{12EI}{h^3} u_o + \frac{6EI}{h^2} \theta_o - \frac{12EI}{h^3} u_1 + \frac{6EI}{h^2} \theta_1 + S_o u_o \qquad \text{(A38)}$$

and

$$b_{11} = F_{x,o} \tag{A39}$$

For the pinned head pile $b_{13} = b_{31} = b_{33} = 0$. The value of $\theta_o = \theta_{free}$ obtained from this solution is used subsequently for establishing the stiffness coefficients of a fixed or partially restrained pile head.

36. For partially restrained, or fixed, head piles three steps in the solution are required. First, the solution for a pinned head pile is performed to obtain θ_{free} . Then, to establish the b_{11} and b_{31} terms of the stiffness matrix, conditions $u_o = 1$ and $\theta_o = (1 - DF)\theta_{free}$ are imposed at the pile head. These conditions result in altering Equations A36 (and A28) to

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} u_o \\ \theta_o \end{Bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} u_1 \\ \theta_1 \end{Bmatrix} \begin{Bmatrix} 1 \\ (1 - DF)\theta_{free} \end{Bmatrix} \tag{A40}$$

The displacements from solution of Equations A34, A35, and A40, together with Equations A28 and A32 for $i = 0$, yield b_{11} as in Equations A38 and A39, and

$$M_{y,o} = \frac{6EI}{h^2} u_o + \frac{4EI}{h} \theta_o - \frac{6EI}{h^2} u_1 + \frac{2EI}{h} \theta_1 \tag{A41}$$

and

$$b_{31} = M_{y,o} \tag{A42}$$

Finally, for b_{13} and b_{31} , conditions $u_o = 0$ and $\theta_o = (DF)\theta_{free}$ are imposed which result in the following form of Equations A36 (and A28):

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{Bmatrix} u_c \\ \theta_c \end{Bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} u_1 \\ \theta_1 \end{Bmatrix} = \begin{Bmatrix} 0 \\ (DF)\cdot\theta_{free} \end{Bmatrix} \tag{A43}$$

Displacements from solution of Equations A34, A36, and A43 with Equations A28, A32, A38, and A41 yield

$$b_{13} = F_{x,c}$$

and

$$b_{33} = M_{y,c}$$

37. The preceding sequence of operations completes the determination of the pile head stiffness matrix for a two-dimensional system in the x-z plane where I in all equations is the moment of inertia of the cross section about the y axis (i.e., $I = I_y$).

38. For a three-dimensional system the pile head force displacement relation is expanded to

$$\begin{Bmatrix} F_x \\ F_y \\ F_z \\ M_x \\ M_y \\ M_z \end{Bmatrix} = \begin{bmatrix} b_{11} & 0 & 0 & 0 & b_{15} & 0 \\ 0 & b_{22} & 0 & b_{24} & 0 & 0 \\ 0 & 0 & b_{33}{}^{*} & 0 & 0 & 0 \\ 0 & b_{42} & 0 & b_{44} & 0 & 0 \\ b_{51} & 0 & 0 & 0 & b_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & b_{66}{}^{*} \end{bmatrix} \begin{Bmatrix} u \\ v \\ w \\ \phi \\ \theta \\ \rho \end{Bmatrix} \tag{A44}$$

---

* b_{33} and b_{66} are coefficients related to axial and torsional effects, respectively, and their determination is not the subject of this appendix.

In this expression coefficients b_{11} , b_{51} , b_{15} , b_{55} represent effects due to displacements, u and θ , in the x-z plane, and are obtained as described in paragraphs 30 through 36 with $I = I_y$. Coefficients b_{22} , b_{42} , b_{24} , and b_{44} , which are related to displacements v and ϕ , in the y-z plane, may also be obtained from the two-dimensional analysis with $I = I_x$ (moment of inertia of the cross section about the x-x axis).

39. This procedure has been expanded to a layered soil system. The continuous system and the finite element model are shown in Figure A4a and A4b.

40. The stiffness S_i must be established from the properties of the surrounding soil. For any node i in layer 1, the soil modulus is

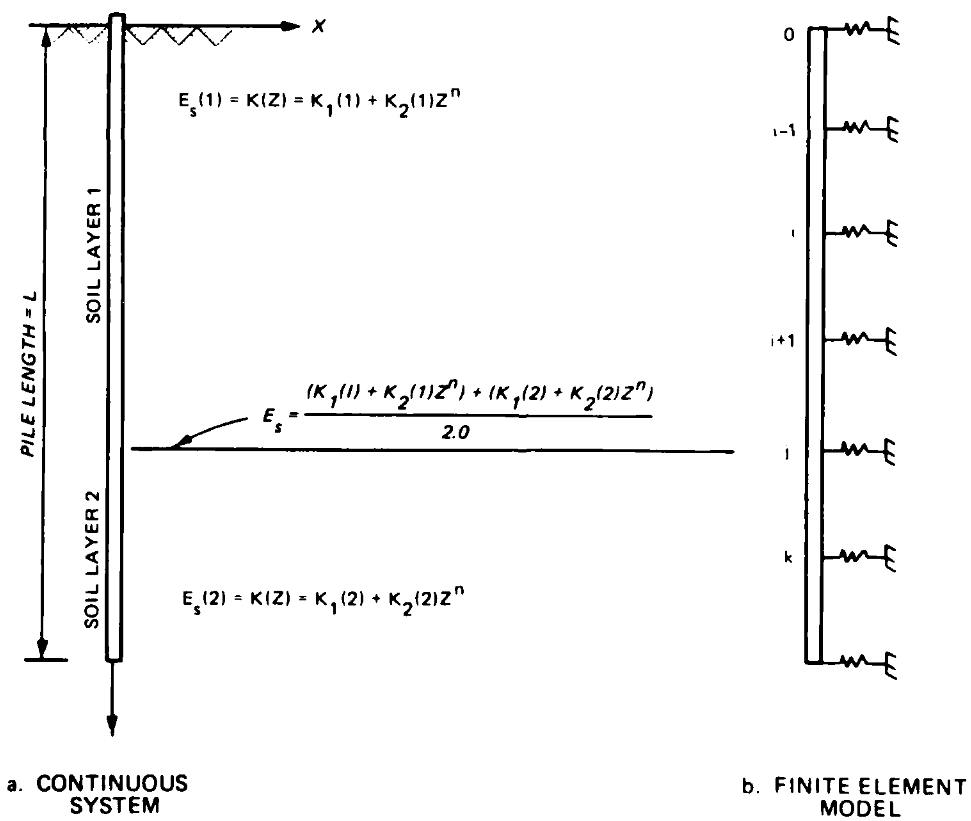

Figure A4. Finite element model of pile-multilayered soil system

$$E_s = K(Z)_i = K_1\ (1) + K_2\ (1)\ Z^n$$

where

$K_1(I)$ = the K_1 constant for layer I

$K_2(I)$ = the K_2 constant for layer I

Z is a length factor that represents the confinement effect. Z will be the depth from the top of the layer to the point if only one layer is present. If more than one layer is present, $Z = Z_{eff}$ where Z_{eff} is the effective depth assuming that the layer properties under consideration extend all the way to the top of the pile. Z_{eff} is dependent on the ratios of the unit weights of the layers and the depth of the overlying layers. Z is always measured from the top of the pile, not the top of the layer. If node j is located at a soil layer (i.e., layers 1 and 2), the soil modulus is

$$E_s = K(Z)_i = \frac{\left(K_1(1) + K_2(1)Z^n_{eff}\right) + \left(K_1(2) + K_2(2)Z^n_{eff}\right)}{2.0}$$

For any node K in any layer M, the soil modulus is

$$K(Z)_k = K_1(M) + K_2(M)Z^n_{eff}$$

Guide to Data Input

41. Data should be input to program PILESTF according to the following guide. All input is free field (a comma or at least one blank should separate data items).

GROUP 1 - Title

IHEAD

IHEAD = 60 Character Problem Heading

GROUP 2 - Pile Properties

A. E, G, XL, DF, NDIM

E = Modulus of elasticity

G = Shear modulus

XL = Pile length (if length is input as zero, program calculates increment length as h = MIN $12I_x$, $12I_y$ and XL = 200 x h, otherwise h = XL/200)

DF = Degree of fixity, $0 \le DF \le 1$

0 - pinned head

1 - fixed head

NDIM = 2 - two dimensional system

3 - three dimensional system

B. XI, YI, XJ, A, AXCO, TOCO

XI = Moment of inertia about X - axis

0 for two dimensional

YI = Moment of inertia about Y - axis

XJ = Torsional moment of Inertia

0 for two dimensional

A = Cross-sectional area

AXCO = Axial stiffness factor

TOCO = Torsional stiffness factor

0 for two dimensional

GROUP 3 - Soil Properties

A. NLAYER

NLAYER = number of soil layers

B. Necessary only if NLAYER = 1

XK1, XK2, ZN

where E_s = XK1 + XK2 **ZN

C. Necessary only if NLAYER > 1

XK1(I), XK2(I), ZN(I), DEPTH(I), GAMMA(I)

where E_s = XK1(I) + XK2(I) **ZN(I)

DEPTH(I) = Bottom elevation of soil layer I (feet)
GAMMA(I) = Unit weight of soil in layer I

Note: Repeat Group 3-C data NLAYER (number of layers) number of times

Example Solutions

42. A number of solutions have been obtained for the pile parameters shown in Table A1. The pile problems solved herein are intended only to demonstrate the use of the program and to indicate the influence of some problem parameters on the pile head stiffness.

Discussion of Results

43. Stiffness coefficients obtained with the computer program are compared in Table A2 with values obtained by the procedures in the previous work section using data from Reese and Matlock (1956) for "infinitely" long piles. Except for problems 3A, 6, and 9, the difference between values predicted by the program and those obtained as in the previous work section are less than one percent. The differences in problem 3A illustrate the effect of length; the pile in this problem is not "infinitely" long. This is substantiated by the results of problems 3B and 3C. The length calculated by the computer program for problem 3C is only an approximation to render the pile "infinitely" long and no other significance should be attached to this value.

44. Example problem 1-A is the same as problem 1 except 2 layers of soil with the same properties were used instead of just 1 layer. The answers from both example problems are the same. The same situation applies to example problems 2 and 2-A.

45. Problem 10 illustrates the effect of different cross section moments of inertia. The pile-soil parameters were chosen to permit comparison of stiffness coefficients associated with forces and displacements in the x-z plane. Data are not available in Reese and Matlock (1956) to permit comparisons for coefficients related to the y-z plane.

Conclusions

46. The example solutions demonstrate the capabilities of the computer program to develop the pile head stiffness matrix for lateral effects of a linearly elastic pile-soil system. The program can readily be extended to permit analysis of piles having variable cross section properties. If procedures were available to approximate the resistance of the soil to axial and/or torsional displacements of the pile, the numerical analysis procedure used in the program could be extended to include these effects as well.

Table A1

Pile-Soil Parameters for Example Solutions

Prob.	NDIM	E (psi)	G (psi)	XL (in.)	DF	IX ($in.^4$)	IY ($in.^4$)	J ($in.^4$)	A ($in.^2$)	AXCO	TOCO	K1 (psi)	K2 $(psi)/(in.)^{ZN}$	ZN
1	2	4.3×10^6	--	1200	0	--	833.33	--	100	1	--	10	0	0
2	2	4.3×10^6	--	1200	1	--	833.33	--	100	1	--	0	10	1
3A	2	1.5×10^6	--	360	0	--	322.06	--	63.5	0.5	--	3.123	0	0
3B	2	1.5×10^6	--	1200	0	--	322.06	--	63.5	0.5	--	3.123	0	0
3C	2	1.5×10^6	--	788.5*	0	--	322.06	--	63.5	0.5	--	3.123	0	0
4	2	1.5×10^6	--	360	0	--	322.06	--	63.5	0.5	--	31.230	0	0
5	2	1.5×10^6	--	360	0	--	322.06	--	63.5	0.5	--	312.300	0	0
6	3	4.3×10^6	1.8×10^6	1200	0	833.33	833.33	1666.67	100	1	1	0	10	1
7	3	4.3×10^6	1.8×10^6	1200	1	833.33	833.33	1666.67	100	1	1	0	10	1
8	3	4.3×10^6	1.8×10^6	1200	0.5	833.33	833.33	1666.67	100	1	1	0	10	1
9	3	4.3×10^6	1.8×10^6	1200	1	833.33	833.33	1666.67	100	1	1	0	10	2
10	2	10×10^6	2.5×10^6	1200	1	1440	1000	2440	120	1	1	160	32	1

* Calculated by program, LENGTH input as zero.

Table A2

Comparison of Results

Prob.	b_{11} for 2D and/or b_{22} for 3D (lb/in.) (1): Program	R&M (2)	% Diff. (3)	(b_{13},b_{31}) for 2D and/or $(b_{15},b_{51},b_{24},b_{42})$ for 3D (lb)(1): Program	R&M (2)	% Diff. (3)	b_{33} for 2D or (b_{44},b_{55}) for 3D (lb-in.) (1): Program	R&M (2)	% Diff. (3)
1	9.732×10^2	9.729×10^2	0.12	--	--	--	--	--	--
2	2.849×10^4	2.843×10^4	0.53	1.356×10^6	1.356×10^6	0	1.045×10^8	1.045×10^8	0
3A	2.306×10^2	2.463×10^2	-6.37	--	--	--	--	--	--
3B	2.464×10^2	2.463×10^2	0.20	--	--	--	--	--	--
3C	2.465×10^2	2.463×10^2	0.04	--	--	--	--	--	--
4	1.384×10^3	1.385×10^3	0	--	--	--	--	--	--
5	7.791×10^3	7.788×10^3	0.17	--	--	--	--	--	--
6	1.090×10^4	1.084×10^4	1.38	--	--	--	--	--	--
7	2.849×10^4	2.843×10^4	0.53	1.356×10^6	1.356×10^6	0	1.045×10^8	1.356×10^6	0
8	1.969×10^4	1.964×10^4	0.71	6.779×10^5	6.778×10^5	0.1	5.226×10^7	5.226×10^7	0
9	2.046×10^5	2.014×10^5	6.06	5.393×10^6	5.356×10^6	2.32	2.143×10^8	2.137×10^8	0.89
10	9.424×10^4 (b_{11})	9.381×10^4	0.76	4.162×10^6 (b_{51})	4.148×10^6	0.29	4.074×10^8 (b_{44})	(4)	--
	1.084×10^5 (b_{22})	(4)	--	-5.165×10^6 (b_{42})	(4)	--	3.046×10^8 (b_{55})	3.039×10^8	0.23

(1) For 3D pile with IX = IY: $b_{11} = b_{22}$, $b_{15} = b_{51} = -b_{42} = -b_{24}$, $b_{44} = b_{55}$.

(2) Coefficients by procedures of Part II with data from Reese and Matlock (1956).

(3) % Diff. = (Program Value - R&M)/R&M*100.

(4) Data are not available in Reese and Matlock (1956) for these values.

```
    ENTER HEADING (60 CHARACTERS MAX.)
=PROB. 1 - 2D, CONSTANT SOIL MODULUS, PINNED HEAD

    ENTER PILE DATA UNDER HEADINGS
       E           G          LENGTH     FIXITY    NDIM
    --------    --------    --------    ------      -
=     4.3D6        0          1200         0         2

       IX          IY          J           A       AXCD   TOCD
    --------    --------    --------    --------   ----   ----
=      0         833.33        0         100        1      0

    INPUT THE NUMBER OF SOIL LAYERS
    NLAYER
   --------
=      1

    ENTER SOIL DATA UNDER HEADINGS
    FOR SOIL MODULUS ES=K1+K2*Z**ZN
    K1          K2          ZN
  --------    --------    --------
=   10          0           0

    OUTPUT FROM PLSTF
    HEADING
    PROB. 1 - 2D, CONSTANT SOIL MODULUS, PINNED HEAD

    PILE DATA
     E              G              LENGTH      FIXITY    NDIM
  4.300D 06    0.              1.200D 03      0.         2
     IX              IY              J               A        AXCD   TOCD
  0.             8.333D 02    0.            1.000D 02 1.000 0.

    SOIL DATA

    NUMBER OF SOIL LAYERS
    NLAYER
       1
    K1          K2          ZN
 1.000D 01  0.          0.

    PILE HEAD STIFFNESS MATRIX FOR 2-D PILE

    9.7317D 02      0.              0.
    0.              3.5833D 05      0.
    0.              0.              0.

    DO YOU WANT ANOTHER RUN? (1=YES,0=NO)
=1
```

```
    ENTER HEADING (60 CHARACTERS MAX.)
=PROB. 1A - 2D, CONSTANT SOIL MODULUS, 2 SOIL LAYERS

    ENTER PILE DATA UNDER HEADINGS
       E          G          LENGTH     FIXITY   NDIM
    --------   --------   --------    ------     -
=     4.3D6       0          1200         0        2

       IX         IY          J           A       AXCO  TOCO
    --------   --------   --------   --------   ----  ----
=       0      833.33        0         100       1      0

    INPUT THE NUMBER OF SOIL LAYERS
    NLAYER
   --------
=     2

    ENTER SOIL DATA UNDER HEADINGS
    FOR SOIL MODULUS ES=K1+K2*Z**ZN
    K1         K2         ZN         DEPTH     GAMMA
  --------   --------   --------   --------   --------
=   10          0          0          600        50
=   10          0          0         1200        50

    OUTPUT FROM PLSTF
    HEADING
    PROB. 1A - 2D, CONSTANT SOIL MODULUS, 2 SOIL LAYERS

    PILE DATA
     E            G              LENGTH      FIXITY   NDIM
  4.300D 06    0.             1.200D 03      0.        2
     IX            IY             J             A        AXCO  TOCO
  0.          8.333D 02    0.           1.000D 02 1.000 0.

    SOIL DATA

    NUMBER OF SOIL LAYERS
    NLAYER
       2
    K1          K2          ZN          DEPTH     GAMMA
 1.000D 01 0.          0.         6.000D 02 5.000D 01
 1.000D 01 0.          0.         1.200D 03 5.000D 01

    PILE HEAD STIFFNESS MATRIX FOR 2-D PILE

    9.7317D 02      0.              0.
    0.              3.5833D 05      0.
    0.              0.              0.

    DO YOU WANT ANOTHER RUN? (1=YES,0=NO)
=1
```

```
    ENTER HEADING (60 CHARACTERS MAX.)
=PROB.2 - 2D, LINEAR SOIL MODULUS, FIXED HEAD

    ENTER PILE DATA UNDER HEADINGS
       E          G         LENGTH     FIXITY    NDIM
    --------   --------   --------    ------      -
=    4.3D6         0        1200         1          2

       IX         IY          J          A       AXCO  TOCO
    --------   --------   --------   --------    ----  ----
=       0       833.33        0         100        1     0

    INPUT THE NUMBER OF SOIL LAYERS
    NLAYER
   --------
=     1

    ENTER SOIL DATA UNDER HEADINGS
    FOR SOIL MODULUS ES=K1+K2*Z**ZN
    K1          K2          ZN
  --------   --------   --------
=    0          10          1

    OUTPUT FROM PLSTF
    HEADING
    PROB.2 - 2D, LINEAR SOIL MODULUS, FIXED HEAD

    PILE DATA
     E              G              LENGTH      FIXITY    NDIM
  4.300D 06    0.              1.200D 03      1.000       2
     IX             IY             J             A        AXCO  TOCO
  0.           8.333D 02    0.             1.000D 02 1.000 0.

    SOIL DATA

    NUMBER OF SOIL LAYERS
    NLAYER
       1
    K1           K2          ZN
 0.          1.000D 01 1.000D 00

    PILE HEAD STIFFNESS MATRIX FOR 2-D PILE

    2.8490D 04      0.              1.3558D 06
    0.              3.5833D 05      0.
    1.3558D 06      0.              1.0451D 08

    DO YOU WANT ANOTHER RUN? (1=YES,0=NO)
=1
```

```
    ENTER HEADING (60 CHARACTERS MAX.)
=PROB. 2A - 2D, LINEAR SOIL MODULUS, 2 SOIL LAYERS

    ENTER PILE DATA UNDER HEADINGS
       E          G        LENGTH     FIXITY    NDIM
    --------   --------   --------   ------      -
=     4.3D6       0         1200        1         2

       IX         IY          J          A       AXCO  TOCO
    --------   --------   --------   --------   ----  ----
=       0       833.33       0         100        1     0

    INPUT THE NUMBER OF SOIL LAYERS
    NLAYER
   --------
=     2

    ENTER SOIL DATA UNDER HEADINGS
    FOR SOIL MODULUS ES=K1+K2*Z**ZN
    K1         K2         ZN         DEPTH      GAMMA
  --------   --------   --------   --------   --------
=    0         10          1          600        50
=    0         10          1         1200 .      50

    OUTPUT FROM PLSTF
    HEADING
    PROB. 2A - 2D, LINEAR SOIL MODULUS, 2 SOIL LAYERS

    PILE DATA
      E            G            LENGTH      FIXITY    NDIM
  4.300D 06    0.            1.200D 03     1.000       2
     IX            IY            J            A        AXCO  TOCO
  0.           8.333D 02    0.           1.000D 02 1.000 0.

    SOIL DATA

    NUMBER OF SOIL LAYERS
    NLAYER
       2
    K1           K2          ZN          DEPTH      GAMMA
 0.          1.000D 01 1.000D 00 6.000D 02 5.000D 01
 0.          1.000D 01 1.000D 00 1.200D 03 5.000D 01

    PILE HEAD STIFFNESS MATRIX FOR 2-D PILE

    2.8490D 04     0.              1.3558D 06
    0.             3.5833D 05      0.
    1.3558D 06     0.              1.0451D 08

    DO YOU WANT ANOTHER RUN? (1=YES,0=NO)
=1
```

```
    ENTER HEADING (60 CHARACTERS MAX.)
=PROB. 3A - 2D, CONST."SOFT" SOIL MOD., "SHORT" PILE

    ENTER PILE DATA UNDER HEADINGS
       E          G         LENGTH     FIXITY    NDIM
    --------   --------   --------   -------     -
=     1.5D6       0          360        0         2

       IX         IY          J          A       AXLD  TOLD
    --------   --------   --------   --------   ----  ----
=        0      322.06        0        63.5      0.5    0

    INPUT THE NUMBER OF SOIL LAYERS
    NLAYER
   --------
=      1

    ENTER SOIL DATA UNDER HEADINGS
    FOR SOIL MODULUS ES=K1+K2*Z**ZN
    K1          K2         ZN
  --------   --------   --------
=  3.123        0          0

    OUTPUT FROM PLSTF
    HEADING
    PROB. 3A - 2D, CONST."SOFT" SOIL MOD., "SHORT" PILE

    PILE DATA
     E            G             LENGTH      FIXITY    NDIM
  1.500D 06    0.            3.600D 02      0.          2
     IX           IY             J             A         AXLD  TOLD
  0.           3.221D 02    0.            6.350D 01 0.500 0.

    SOIL DATA

    NUMBER OF SOIL LAYERS
    NLAYER
       1
    K1          K2          ZN
 3.123D 00 0.          0.

    PILE HEAD STIFFNESS MATRIX FOR 2-D PILE

    2.3058D 02      0.              0.
    0.              1.3229D 05      0.
    0.              0.              0.

    DO YOU WANT ANOTHER RUN? (1=YES,0=NO)
=1
```

```
    ENTER HEADING (60 CHARACTERS MAX.)
=PROB. 3B - SAME AS 3A EXCEPT "LONG" PILE

    ENTER PILE DATA UNDER HEADINGS
       E           G           LENGTH      FIXITY    NDIM
    --------    --------    --------    ------      -
=     1.5D6        0         1200          0          2

       IX          IY          J           A         AXCO  TOCO
    --------    --------    --------    --------    ----  ----
=       0        322.06        0          63.5       0.5    0

    INPUT THE NUMBER OF SOIL LAYERS
    NLAYER
   --------
=      1

    ENTER SOIL DATA UNDER HEADINGS
    FOR SOIL MODULUS ES=K1+K2*Z**ZN
    K1          K2          ZN
  --------    --------    --------
=  3.123        0           0

    OUTPUT FROM PLSTF
    HEADING
    PROB. 3B - SAME AS 3A EXCEPT "LONG" PILE

    PILE DATA
     E             G             LENGTH       FIXITY    NDIM
  1.500D 06    0.             1.200D 03      0.          2
     IX            IY             J             A         AXCO   TOCO
  0.            3.221D 02    0.             6.350D 01 0.500 0.

    SOIL DATA

    NUMBER OF SOIL LAYERS
    NLAYER
       1
    K1          K2           ZN
 3.123D 00 0.          0.

    PILE HEAD STIFFNESS MATRIX FOR 2-D PILE

    2.4639D 02      0.              0.
    0.              3.9688D 04      0.
    0.              0.              0.

    DO YOU WANT ANOTHER RUN? (1=YES,0=NO)
=1
```

```
    ENTER HEADING (60 CHARACTERS MAX.)
=PROB. 3C - SAME AS 3A EXCEPT LENGTH CALC. BY PROG.

    ENTER PILE DATA UNDER HEADINGS
        E         G          LENGTH      FIXITY    NDIM
    --------  --------   --------     ------      -
=     1.5D6        0          0            0          2

        IX        IY          J            A        AXCD   TOCD
    --------  --------   --------   ---------   ----   ----
=        0      322.06        0          63.5       0.5     0

    INPUT THE NUMBER OF SOIL LAYERS
    NLAYER
   --------
=      1

    ENTER SOIL DATA UNDER HEADINGS
    FOR SOIL MODULUS ES=K1+K2*Z**ZN
    K1         K2         ZN
  --------  --------  --------
=  3.123        0          0

    OUTPUT FROM PLSTF
    HEADING
    PROB. 3C - SAME AS 3A EXCEPT LENGTH CALC. BY PROG.

    PILE DATA
     E              G             LENGTH      FIXITY    NDIM
  1.500D 06    0.             1.577D 03      0.         2
     IX            IY             J             A        AXCD  TOCD
  0.            3.221D 02    0.           6.350D 01 0.500 0.

    SOIL DATA

    NUMBER OF SOIL LAYERS
    NLAYER
       1
    K1         K2         ZN
 3.123D 00 0.          0.

    PILE HEAD STIFFNESS MATRIX FOR 2-D PILE

    2.4648D 02      0.             0.
    0.              3.0201D 04     0.
    0.              0.             0.

    DO YOU WANT ANOTHER RUN? (1=YES,0=NO)
=1
```

```
    ENTER HEADING (60 CHARACTERS MAX.)
=PROB. 4 - SAME AS 3A EXCEPT STIFFER SOIL

    ENTER PILE DATA UNDER HEADINGS
    -  E          G          LENGTH      FIXITY    NDIM
    --------   --------   --------    ------     -
=     1.5D6        0          360          0         2

       IX         IY          J          A         AXCO   TOCO
    --------   --------   --------   --------   ----   ----
=        0      322.06        0         63.5       0.5     0

    INPUT THE NUMBER OF SOIL LAYERS
    NLAYER
   --------
=      1

    ENTER SOIL DATA UNDER HEADINGS
    FOR SOIL MODULUS ES=K1+K2*Z**ZN
    K1          K2          ZN
  --------   --------   --------
=  31.23         0          0

    OUTPUT FROM PLSTF
    HEADING
    PROB. 4 - SAME AS 3A EXCEPT STIFFER SOIL

    PILE DATA
     E             G             LENGTH      FIXITY    NDIM
  1.500D 06    0.            3.600D 02      0.         2
     IX            IY             J             A         AXCO   TOCO
  0.           3.221D 02    0.            6.350D 01  0.500  0.

    SOIL DATA

    NUMBER OF SOIL LAYERS
    NLAYER
       1
    K1          K2          ZN
 3.123D 01  0.          0.

    PILE HEAD STIFFNESS MATRIX FOR 2-D PILE

    1.3840D 03      0.              0.
    0.              1.3229D 05      0.
    0.              0.              0.

    DO YOU WANT ANOTHER RUN? (1=YES,0=NO)
=1
```

```
    ENTER HEADING (60 CHARACTERS MAX.)
=PROB. 5 - SAME AS 3A AND 4 EXCEPT STIFFER SOIL

    ENTER PILE DATA UNDER HEADINGS
       E          G        LENGTH     FIXITY   NDIM
    --------   --------   --------   ------     -
=     1.5D6       0         360         0        2

       IX         IY         J          A      AXCO  TOCO
    --------   --------   --------   --------  ----  ----
=       0       322.06       0        63.5     0.5     0

    INPUT THE NUMBER OF SOIL LAYERS
    NLAYER
   --------
=      1

    ENTER SOIL DATA UNDER HEADINGS
    FOR SOIL MODULUS ES=K1+K2*Z**ZN
    K1         K2        ZN
  --------   --------   --------
=  312.3        0          0

    OUTPUT FROM PLSTF
    HEADING
    PROB. 5 - SAME AS 3A AND 4 EXCEPT STIFFER SOIL

    PILE DATA
     E             G             LENGTH      FIXITY    NDIM
  1.500D 06    0.            3.600D 02      0.          2
     IX             IY            J              A       AXCO   TOCO
  0.           3.221D 02    0.           6.350D 01  0.500  0.

    SOIL DATA

    NUMBER OF SOIL LAYERS
    NLAYER
       1
    K1          K2          ZN
 3.123D 02  0.          0.

    PILE HEAD STIFFNESS MATRIX FOR 2-D PILE

    7.7913D 03      0.              0.
    0.              1.3229D 05      0.
    0.              0.              0.

    DO YOU WANT ANOTHER RUN? (1=YES,0=NO)
=1
```

```
    ENTER HEADING (60 CHARACTERS MAX.)
=PROB. 6 - 3D, LINEAR SOIL MODULUS,  PINNED HEAD

    ENTER PILE DATA UNDER HEADINGS
       E          G          LENGTH     FIXITY    NDIM
    --------   --------   --------    ------      -
=     4.3D6      1.8D6      1200         0         3

       IX         IY          J          H       HXCD  TOCD
    --------   --------   --------  --------   ----  ----
=     833.33     833.33    1666.67     100        1     1

    INPUT THE NUMBER OF SOIL LAYERS
    NLAYER
  --------
=      1

    ENTER SOIL DATA UNDER HEADINGS
    FOR SOIL MODULUS ES=K1+K2*Z**ZN
    K1         K2         ZN
  --------   --------   --------
=     0         10          1

    OUTPUT FROM PLSTF
    HEADING
    PROB. 6 - 3D, LINEAR SOIL MODULUS,  PINNED HEAD

    PILE DATA
     E              G              LENGTH       FIXITY    NDIM
  4.300D 06    1.800D 06    1.200D 03      0.          3
     IX             IY              J            H          HXCD  TOCD
  8.333D 02    8.333D 02    1.667D 03    1.000D 02 1.000 1.000

    SOIL DATA

    NUMBER OF SOIL LAYERS
    NLAYER
       1
    K1             K2             ZN
 0.          1.000D 01 1.000D 00

    PILE HEAD STIFFNESS MATRIX FOR 3-D PILE

 1.0901D 04   0.           0.           0.           0.           0.
 0.           1.0901D 04   0.           0.           0.           0.
 0.           0.           3.5833D 05   0.           0.           0.
 0.           0.           0.           0.           0.           0.
 0.           0.           0.           0.           0.           0.
 0.           0.           0.           0.           0.           2.5000D 06

    DO YOU WANT ANOTHER RUN? (1=YES,0=NO)
=1
```

```
    ENTER HEADING (60 CHARACTERS MAX.)
=PROB. 7 - SAME AS 6 EXCEPT FIXED HEAD

    ENTER PILE DATA UNDER HEADINGS
        E          G        LENGTH     FIXITY    NDIM
    --------   --------   --------    ------      -
=     4.3D6      1.8D6      1200         1        3

       IX         IY          J          H       HXCO  TOCO
    --------   --------   --------   --------    ----  ----
=     833.33     833.33     1666.67    100         1     1

    INPUT THE NUMBER OF SOIL LAYERS
    NLAYER
   --------
=      1

    ENTER SOIL DATA UNDER HEADINGS
    FOR SOIL MODULUS ES=K1+K2*Z**ZN
    K1          K2          ZN
  --------    --------    --------
=     0          10          1

    OUTPUT FROM PLSTF
    HEADING
    PROB. 7 - SAME AS 6 EXCEPT FIXED HEAD

    PILE DATA
     E              G              LENGTH       FIXITY     NDIM
  4.300D 06     1.800D 06     1.200D 03      1.000        3
     IX             IY               J              H         HXCO   TOCO
  8.333D 02     8.333D 02     1.667D 03     1.000D 02 1.000 1.000

    SOIL DATA

    NUMBER OF SOIL LAYERS
    NLAYER
       1
    K1             K2           ZN
 0.           1.000D 01 1.000D 00

    PILE HEAD STIFFNESS MATRIX FOR 3-D PILE

 2.8490D 04   0.             0.             0.             1.3558D 06   0.
 0.           2.8490D 04     0.            -1.3558D 06     0.           0.
 0.           0.             3.5833D 05     0.             0.           0.
 0.          -1.3558D 06     0.             1.0451D 08     0.           0.
 1.3558D 06   0.             0.             0.             1.0451D 08   0.
 0.           0.             0.             0.             0.           2.5000D 06

    DO YOU WANT ANOTHER RUN? (1=YES,0=NO)
=1
```

```
    ENTER HEADING (60 CHARACTERS MAX.)
=PROB. 8 - SAME AS 7 EXCEPT PARTIALLY RESTRAINED HEAD

    ENTER PILE DATA UNDER HEADINGS
       E          G        LENGTH    FIXITY   NDIM
    --------   --------   --------   ------    -
=    4.3D6      1.8D6      1200        0.5      3

       IX         IY          J         A      AXCO  TOCO
    --------   --------   --------  --------   ----  ----
=    833.33     833.33    1666.67    100        1     1

    INPUT THE NUMBER OF SOIL LAYERS
    NLAYER
   --------
=     1

    ENTER SOIL DATA UNDER HEADINGS
    FOR SOIL MODULUS ES=K1+K2*Z**ZN
    K1         K2         ZN
  --------   --------   --------
=    0          10         1

    OUTPUT FROM PLSTF
    HEADING
    PROB. 8 - SAME AS 7 EXCEPT PARTIALLY RESTRAINED HEAD

    PILE DATA
     E              G            LENGTH      FIXITY    NDIM
  4.300D 06    1.800D 06    1.200D 03     0.500       3
     IX             IY            J            A       AXCO  TOCO
  8.333D 02    8.333D 02    1.667D 03    1.000D 02 1.000 1.000

    SOIL DATA

    NUMBER OF SOIL LAYERS
    NLAYER
       1
    K1          K2          ZN
 0.          1.000D 01 1.000D 00

    PILE HEAD STIFFNESS MATRIX FOR 3-D PILE

 1.9695D 04  0.          0.           0.           6.7791D 05  0.
 0.          1.9695D 04  0.          -6.7791D 05   0.          0.
 0.          0.          3.5833D 05   0.           0.          0.
 0.         -6.7791D 05  0.           5.2256D 07   0.          0.
 6.7791D 05  0.          0.           0.           5.2256D 07  0.
 0.          0.          0.           0.           0.          2.5000D 06

    DO YOU WANT ANOTHER RUN? (1=YES,0=NO)
=1
```

```
    ENTER HEADING (60 CHARACTERS MAX.)
=PROB. 9 - SAME AS 7 EXCEPT EXPONENTIAL SOIL MODULUS

    ENTER PILE DATA UNDER HEADINGS
        E          G         LENGTH      FIXITY    NDIM
    --------   --------   --------    ------      -
=     4.3D6      1.8D6      1200          1          3

       IX         IY          J           A        AXCO  TOCO
    --------   --------   --------   --------    ----  ----
=     833.33     833.33     1666.67     100         1      1

    INPUT THE NUMBER OF SOIL LAYERS
    NLAYER
   --------
=      1

    ENTER SOIL DATA UNDER HEADINGS
    FOR SOIL MODULUS ES=K1+K2*Z**ZN
    K1          K2          ZN
  --------   --------   --------
=     0          10          2

    OUTPUT FROM PLSTF
    HEADING
    PROB. 9 - SAME AS 7 EXCEPT EXPONENTIAL SOIL MODULUS

    PILE DATA
     E             G             LENGTH      FIXITY    NDIM
  4.300D 06    1.800D 06    1.200D 03      1.000       3
     IX            IY            J             A        AXCO  TOCO
  8.333D 02    8.333D 02    1.667D 03    1.000D 02 1.000 1.000

    SOIL DATA

    NUMBER OF SOIL LAYERS
    NLAYER
       1
    K1            K2          ZN
 0.          1.000D 01 2.000D 00

    PILE HEAD STIFFNESS MATRIX FOR 3-D PILE

 2.0465D 05   0.           0.           0.            5.3927D 06   0.
 0.           2.0465D 05   0.          -5.3927D 06    0.           0.
 0.           0.           3.5833D 05   0.            0.           0.
 0.          -5.3927D 06   0.           2.1433D 08    0.           0.
 5.3927D 06   0.           0.           0.            2.1433D 08   0.
 0.           0.           0.           0.            0.           2.5000D 06

    DO YOU WANT ANOTHER RUN? (1=YES,0=NO)
=1
```

```
    ENTER HEADING (60 CHARACTERS MAX.)
=PROB. 10 - 3D, DIFF. IX & IY, LINEAR SOIL MODULUS

    ENTER PILE DATA UNDER HEADINGS
       E          G          LENGTH     FIXITY    NDIM
    --------   --------   --------    ------     -
=     10.D6      2.5D6      1200        1          3

       IX         IY          J          A        AXCO   TOCO
    --------   --------   --------   --------   ----   ----
=     1440       1000       2440       120         1      1

    INPUT THE NUMBER OF SOIL LAYERS
    NLAYER
   --------
=     1

    ENTER SOIL DATA UNDER HEADINGS
    FOR SOIL MODULUS ES=K1+K2*Z**ZN
    K1          K2          ZN
  --------   --------   --------
=  160         32           1

    OUTPUT FROM PLSTF
    HEADING
    PROB. 10 - 3D, DIFF. IX & IY, LINEAR SOIL MODULUS

    PILE DATA
     E              G              LENGTH        FIXITY    NDIM
  1.000D 07    2.500D 06     1.200D 03       1.000       3
     IX             IY             J              A        AXCO   TOCO
  1.440D 03    1.000D 03     2.440D 03     1.200D 02 1.000 1.000

    SOIL DATA

    NUMBER OF SOIL LAYERS
    NLAYER
       1
    K1          K2          ZN
 1.600D 02 3.200D 01 1.000D 00

    PILE HEAD STIFFNESS MATRIX FOR 3-D PILE

 9.4239D 04   0.           0.            0.           4.1617D 06   0.
 0.           1.0838D 05   0.           -5.1653D 06   0.           0.
 0.           0.           1.0000D 06   0.           0.           0.
 0.          -5.1653D 06   0.            4.0736D 08   0.           0.
 4.1617D 06   0.           0.            0.           3.0462D 08   0.
 0.           0.           0.            0.           0.           5.0833D 06

    DO YOU WANT ANOTHER RUN? (1=YES,0=NO)
=0
```

```
    ENTER HEADING (60 CHARACTERS MAX.)
=PROB. 11 - SAME AS 9 EXCEPT 3 SOIL LAYERS

    ENTER PILE DATA UNDER HEADINGS
       E          G        LENGTH    FIXITY   NDIM
    --------   --------   --------   ------     -
=     4.3D6      1.8D6      1200        1        3

      IX         IY          J          A      AXCO  TOCO
    --------   --------   --------   --------  ----  ----
=     833.33     833.33   1666.67      100       1     1

    INPUT THE NUMBER OF SOIL LAYERS
    NLAYER
   --------
=      3

    ENTER SOIL DATA UNDER HEADINGS
    FOR SOIL MODULUS ES=K1+K2*Z**ZN
    K1          K2          ZN         DEPTH     GAMMA
  --------   --------   --------   --------   --------
=    10          0          0           300       50
=    10          0          0           600       40
=     0         10          1          1200       50

    OUTPUT FROM PLSTF
    HEADING
    PROB. 11 - SAME AS 9 EXCEPT 3 SOIL LAYERS

    PILE DATA
      E            G            LENGTH      FIXITY    NDIM
  4.300D 06    1.800D 06    1.200D 03      1.000      3
     IX           IY            J            A        AXCO  TOCO
  8.333D 02    8.333D 02    1.667D 03    1.000D 02 1.000 1.000

    SOIL DATA

    NUMBER OF SOIL LAYERS
    NLAYER
       3
    K1          K2          ZN          DEPTH      GAMMA
 1.000D 01 0.           0.          3.000D 02 5.000D 01
 1.000D 01 0.           0.          6.000D 02 4.000D 01
 0.          1.000D 01 1.000D 00 1.200D 03 5.000D 01

    PILE HEAD STIFFNESS MATRIX FOR 3-D PILE

 1.9529D 03  0.           0.           0.           1.8931D 05   0.
 0.           1.9529D 03  0.          -1.8931D 05  0.            0.
 0.           0.           3.5833D 05  0.           0.            0.
 0.          -1.8931D 05  0.           3.6911D 07  0.            0.
 1.8931D 05  0.           0.           0.           3.6911D 07   0.
 0.           0.           0.           0.           0.            2.5000D 06

    DO YOU WANT ANOTHER RUN? (1=YES,0=NO)
=0
```

```
    ENTER HEADING (60 CHARACTERS MAX.)
=PROB. 12 - SAME AS 11 EXCEPT DIFF. SOIL PROPERTIES

    ENTER PILE DATA UNDER HEADINGS
       E          G        LENGTH     FIXITY    NDIM
    --------   --------   --------    ------     -
=     4.3D6      1.8D6      1200         1        3

       IX         IY         J          A        AXCO   TOCO
    --------   --------   --------   --------    ----   ----
=     833.33     833.33    1666.67    100         1      1

    INPUT THE NUMBER OF SOIL LAYERS
    NLAYER
   --------
=       3

    ENTER SOIL DATA UNDER HEADINGS
    FOR SOIL MODULUS ES=K1+K2*Z**ZN
    K1          K2          ZN          DEPTH      GAMMA
  --------   --------   --------   --------   --------
=    10          0          0           300        50
=     0         10          1           600        40
=     0         10          2          1200        50

    OUTPUT FROM PLSTF
    HEADING
    PROB. 12 - SAME AS 11 EXCEPT DIFF. SOIL PROPERTIES

    PILE DATA
     E             G            LENGTH      FIXITY    NDIM
  4.300D 06    1.800D 06    1.200D 03     1.000       3
     IX            IY            J             A       AXCO   TOCO
  8.333D 02    8.333D 02    1.667D 03    1.000D 02 1.000 1.000

    SOIL DATA

    NUMBER OF SOIL LAYERS
    NLAYER
       3
    K1          K2          ZN          DEPTH      GAMMA
 1.000D 01 0.          0.         3.000D 02 5.000D 01
 0.         1.000D 01 1.000D 00 6.000D 02 4.000D 01
 0.         1.000D 01 2.000D 00 1.200D 03 5.000D 01

    PILE HEAD STIFFNESS MATRIX FOR 3-D PILE

 2.2926D 03  0.           0.           0.           2.4064D 05  0.
 0.          2.2926D 03   0.          -2.4064D 05   0.           0.
 0.          0.           3.5833D 05   0.           0.           0.
 0.         -2.4064D 05   0.           4.5295D 07   0.           0.
 2.4064D 05  0.           0.           0.           4.5295D 07   0.
 0.          0.           0.           0.           0.           2.5000D 06

    DO YOU WANT ANOTHER RUN? (1=YES,0=NO)
=0
```

APPENDIX B: USER'S GUIDE FOR PROGRAM FDRAW

General Introduction

1. Documentation for the computer program FDRAW--an interactive graphics post-processor--is presented in this appendix and includes a general introduction, guide for data input, and example problems.

2. FDRAW is capable of displaying pile geometry, resultant axial forces, several different pile loading factors, and elastic center diagrams. Program commands control the display. The program was developed by Mr. John Jobst, St. Louis District.

3. FDRAW runs on the WES G-635, Macon H-6000, and Boeing CDC computers in the time-sharing mode. It is limited to execution on a Tektronix 4014. The program is part of the CORPS library and is identified by the program number X0036. To execute the program, issue one of the following run commands. On the WES or Macon computer,

```
OLD WESLIB/CORPS/X0036,R
GCS2D
device - TK4
```

On the Boeing computer,

```
OLD,CORPS/UN=CECELB
CALL,CORPS,X0036
```

4. Two bits of information are prompted for by the program before any commands may be given:

   a. The name of the plotting file created by an analysis run of X0034.

   b. The radius of the figures to be drawn on the screen (pile coordinate units per inch of screen).

5. After this, any valid FDRAW command may be given. The program assumes load case 1 to be the current load case until it is changed by giving the "LOAD" command.

COMMANDS:

HELP	To obtain a list of valid commands.
RADI	To redefine the radius of the figures drawn on the screen (units that the coordinates of the piles are given in per inch of screen).

LOAD To change the current load case.

GEOM To display pile locations, with the options of printing the batter for each battered pile and/or numbering the pile.

COMB To display the combined bending factor for each pile and the portion of that factor due solely to the axial load on the pile, for the current load case.

$$C.B.F. = \frac{Q3}{FA} + \frac{Q4}{FB4} + \frac{Q5}{FB5}$$

where Q3 = vertical load along U_3 axis (kips)

Q4 = moment about U_1 axis (kip-ft)

Q5 = moment about U_2 axis (kip-ft)

FA = allowable axial load (kips)

FB4 = allowable moment about minor principal axis (kip-ft)

FB5 = allowable moment about major principal axis (kip-ft)

PLF To display the pile load factor and P.L.F. Flag for each pile for the current load case.

For pile in compression:

$$P.L.F. = MAX \begin{Bmatrix} C.B.F. \\ A.F. \end{Bmatrix} \quad P.L.F.\ Flag = \begin{Bmatrix} BC \\ C \end{Bmatrix}$$

For pile in tension:

$$P.L.F. = MAX \begin{Bmatrix} C.B.F. \\ A.F. \end{Bmatrix} \quad P.L.F.\ Flag = \begin{Bmatrix} BT \\ T \end{Bmatrix}$$

AXFC To display the axial factor and P.L.F. Flag for each pile for the current load case.

For pile in compression:

$$A.F. = \frac{Q3}{CALOW}$$

Where CALOW = Allowable Compressive Load (kips)
For pile in tension:

$$A.F. = \left| \frac{Q3}{TALOW} \right|$$

where TALOW = Allowable Tensile Load (kips)

FORCE — To display the axial force, Q3, for each pile for the current load case.

CCOMB — Similar to "COMB" except that it displays the critical combined bending factor for all load cases and the critical load case number, for each pile.

CPLF — Similar to "PLF" except that it displays the critical pile load factor for all load cases and the critical load case number, for each pile.

CCAXFC — Similar to "AXFC" except that it displays the critical axial factor for all load cases and the critical load case number, for all pile in compression.

CTAXFC — To display everything "CCAXFC" does, except for all pile in tension.

ELCEN — To display the elastic center and resultant force diagrams for all load cases.

END — To end FDRAW.

Guide for Data Input

6. Program X0034 of the CORPS library creates a plotting file to be used as the input data file for this program. Data are written to this file according to the following guide. All input is in free field (a comma or at least one blank should separate data items) except Group 5.

Group 1 - Pile Data

A. $\boxed{\text{LINE,NP}}$

LINE = five digit line number

NP = total number of piles in the foundation

B. Note: Repeat NP (Number of Piles) number of times

$\boxed{\text{LINE,U1,U2,U3,H,ANG}}$

U1 = distance from origin to pile along U1-axis (feet)

U2 = distance from origin to pile along U2-axis (feet)

U3 = distance from origin to pile along U3-axis (feet)

H = batter H vertical on 1 horizontal 0 -- vertical pile

ANG = clockwise angle between the positive U1 axis of the structure and the U1 axis (direction of batter) of the pile (degrees)

Group 2 - Loading Control Data

LINE,NLDCS

NLDCS - Number of Loading Conditions

Group 3 - Elastic Center Data

LINE,EC1,EC2,EC31,EC32

EC1 - U1 coordinate of elastic center in U1-U3 plane

EC2 - U2 coordinate of elastic center in U2-U3 plane

EC31 - U3 coordinate of elastic center in U2-U3 plane

EC32 - U3 coordinate of elastic center in U2-U3 plane

Group 4 - Applied Loads

LINE,Q1,Q2,Q3,Q5,Q4

Q1 = horizontal load along U_1-axis (kips)

Q2 = horizontal load along U_2-axis (kips)

Q3 = vertical load along U_3-axis (kips)

Q5 = moment about U_2-axis (kip-ft)

Q4 = moment about U_1-axis (kip-ft)

Group 5 - Load Factors

LINE,LC,CBF,PLF,FLAG,AFFC,STFC,Q3

		Cols
LC	= Loading Case	6-8
CBF	= Combined bending factor	9-18
PLF	= Pile load factor	19-28
FLAG	= Flag denoting compression or tension	30-31
	C = compression	
	T = tension	
	BC = compression in combined bending	
	BT = tension in combined bending	
	B = if axial force = 0	
AFFC	= Axial factor	32-41
STFC	= Steel axial factor	42-51
Q3	= Axial force	52-61

[illegible] Group 5 data NP (Number of piles) Number of times.
[illegible] Groups 4 and 5 data NLDCS (number of loading
[illegible] number of times.

Example Problems

7. The examples which follow illustrate the displays available from FDRAW. The data used are the output from example problem 9 presented on pages 101-109 of the main text of this report. The input data are stored in a file and are presented in Table B1. There is 1 load case and 27 piles in the foundation.

Table B1

Input Data for Example Problems

```
10000  27
10010   14.00     1.50     0.       3.00  270.00
10020   10.00     1.50     0.       3.00  270.00
10030    6.00     1.50     0.       3.00  270.00
10040    2.00     1.50     0.       3.00  270.00
10050   -2.00     1.50     0.       3.00  270.00
10060   -6.00     1.50     0.       3.00  270.00
10070  -10.00     1.50     0.       3.00  270.00
10080  -14.00     1.50     0.       3.00  270.00
10090   12.00     4.50     0.       3.00  270.00
10100    4.00     4.50     0.       3.00  270.00
10110   -4.00     4.50     0.       3.00  270.00
10120  -12.00     4.50     0.       3.00  270.00
10130   12.00     9.00     0.       3.00   90.00
10140    4.00     9.00     0.       3.00   90.00
10150   -4.00     9.00     0.       3.00   90.00
10160  -12.00     9.00     0.       3.00   90.00
10170   14.00    12.00     0.       3.00   90.00
10180   10.00    12.00     0.       3.00   90.00
10190    6.00    12.00     0.       3.00   90.00
10200    2.00    12.00     0.       3.00   90.00
10210   -2.00    12.00     0.       3.00   90.00
10220   -6.00    12.00     0.       3.00   90.00
10230  -10.00    12.00     0.       3.00   90.00
10240  -14.00    12.00     0.       3.00   90.00
10250   16.00     3.00     0.       0.      0.
10260   12.00     3.00     0.       0.      0.
10270    8.00     3.00     0.       0.      0.
10280  1
10290          17.535       75.520        0.000     -130.605
10300          0.          276.961      344.900        0.          5287.422
10310  1       0.02       0.02 BT    -0.02      -0.02      -20.40
10320  1       0.02       0.02 BT    -0.02      -0.02      -19.91
10330  1       0.02       0.02 BT    -0.02      -0.02      -19.42
10340  1       0.02       0.02 BT    -0.02      -0.02      -18.92
10350  1       0.02       0.02 BT    -0.02      -0.02      -18.43
10360  1       0.02       0.02 BT    -0.02      -0.02      -17.94
10370  1       0.02       0.02 BT    -0.02      -0.02      -17.45
10380  1       0.02       0.02 BT    -0.02      -0.02      -16.96
10390  1       0.02       0.02 BT    -0.02      -0.02      -18.61
10400  1       0.02       0.02 BT    -0.02      -0.02      -17.63
10410  1       0.02       0.02 BT    -0.02      -0.02      -16.65
10420  1       0.02       0.02 BT    -0.02      -0.02      -15.67
10430  1       0.04       0.04 C      0.04       0.04       41.64
10440  1       0.04       0.04 C      0.04       0.04       43.70
10450  1       0.05       0.05 C      0.05       0.05       45.75
10460  1       0.05       0.05 C      0.05       0.05       47.80
10470  1       0.04       0.04 C      0.04       0.04       42.67
10480  1       0.04       0.04 C      0.04       0.04       43.70
10490  1       0.04       0.04 C      0.04       0.04       44.73
10500  1       0.05       0.05 C      0.05       0.05       45.75
10510  1       0.05       0.05 C      0.05       0.05       46.78
10520  1       0.05       0.05 C      0.05       0.05       47.81
10530  1       0.05       0.05 C      0.05       0.05       48.83
10540  1       0.05       0.05 C      0.05       0.05       42.86
10550  1       0.01       0.01 C      0.01       0.01        9.31
10560  1       0.01       0.01 C      0.01       0.01       10.11
10670  1       0.01       0.01 C      0.01       0.01       10.91

ready

s
```

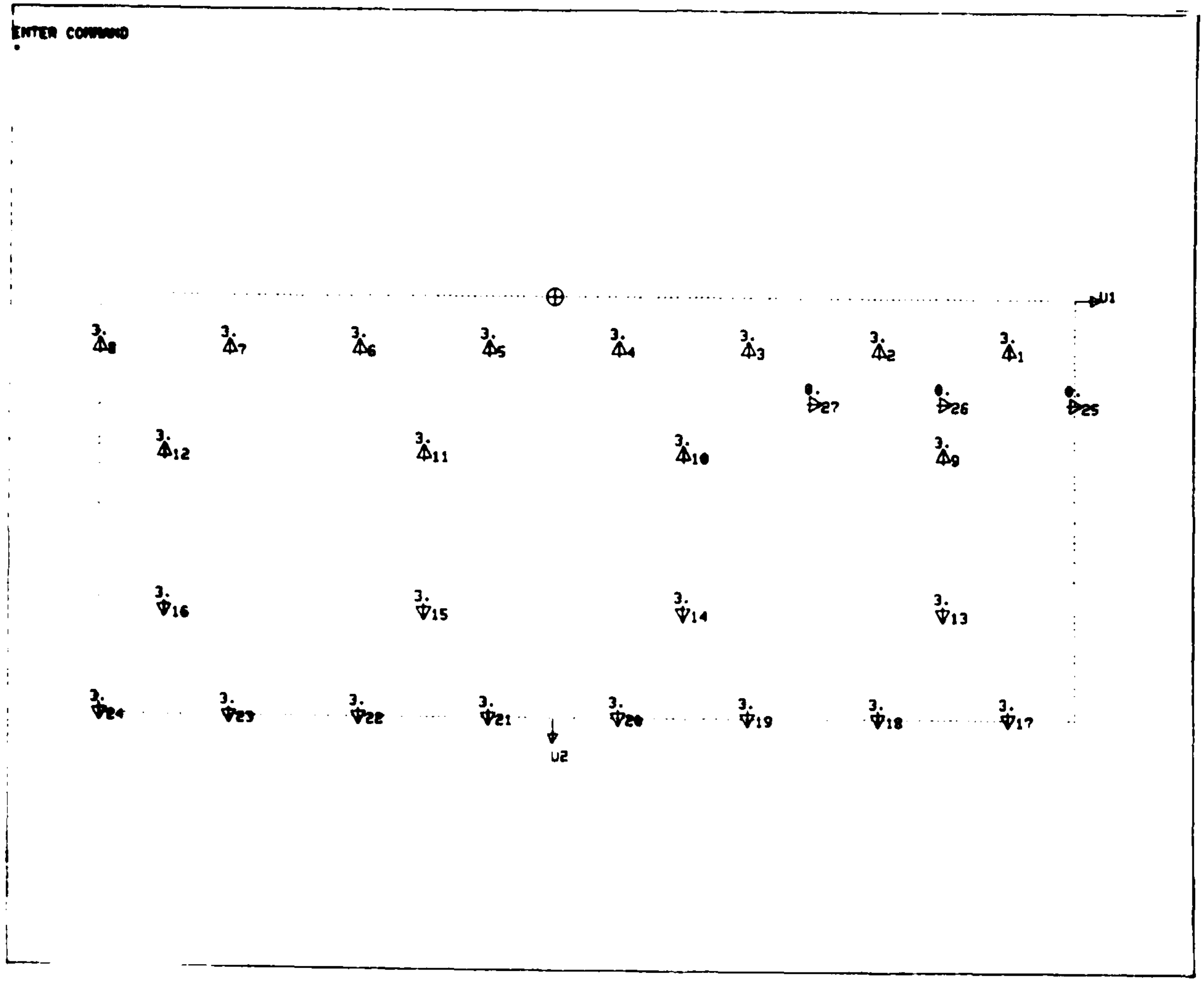

Figure B1. Display of pile numbers, locations, and batter. Enter command GEOM to obtain this diagram

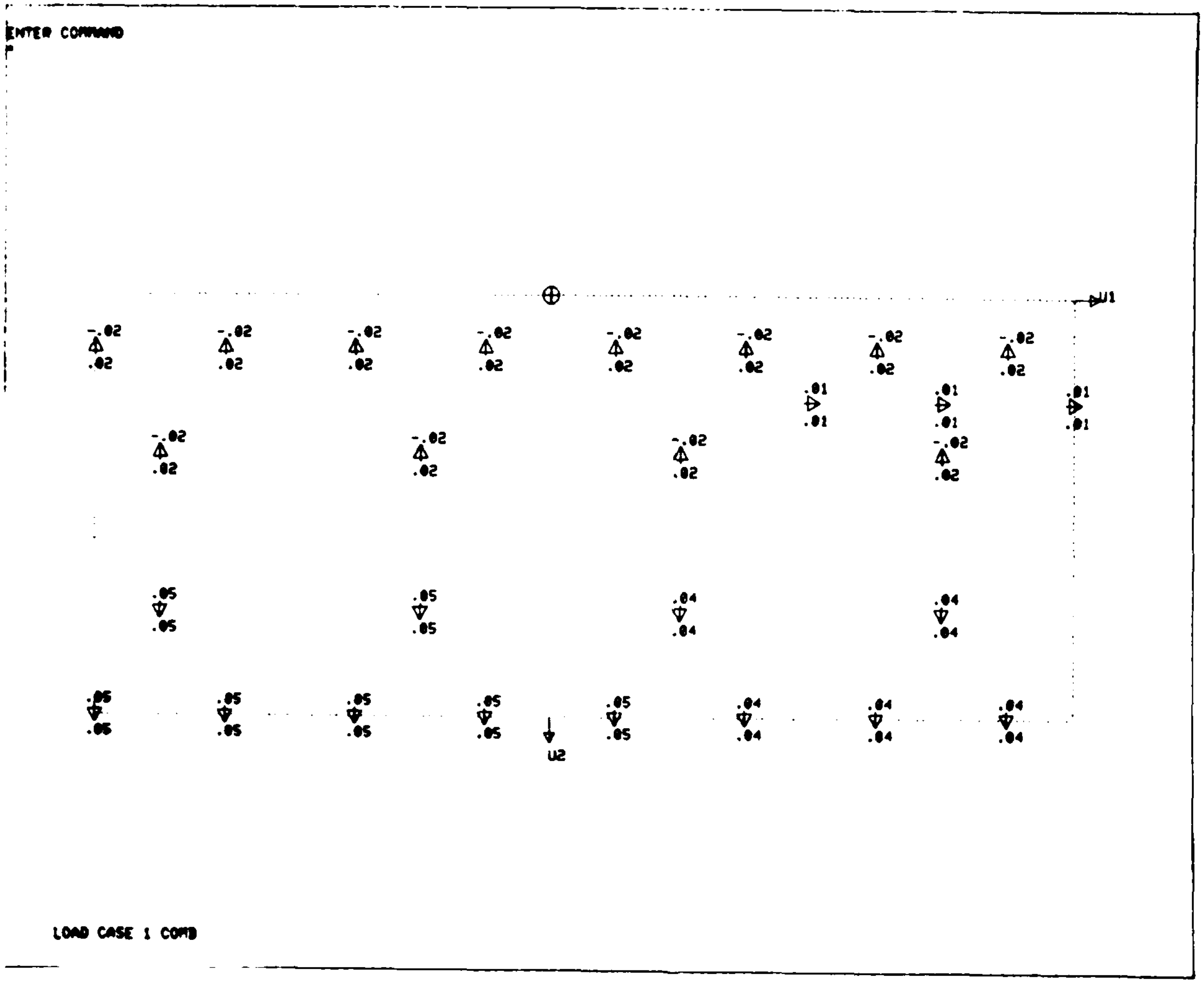

Figure B2. Display of combined bending factor for each pile and the portion of that factor due solely to the axial load on the pile, for the current load case. Enter command COMB to obtain this diagram

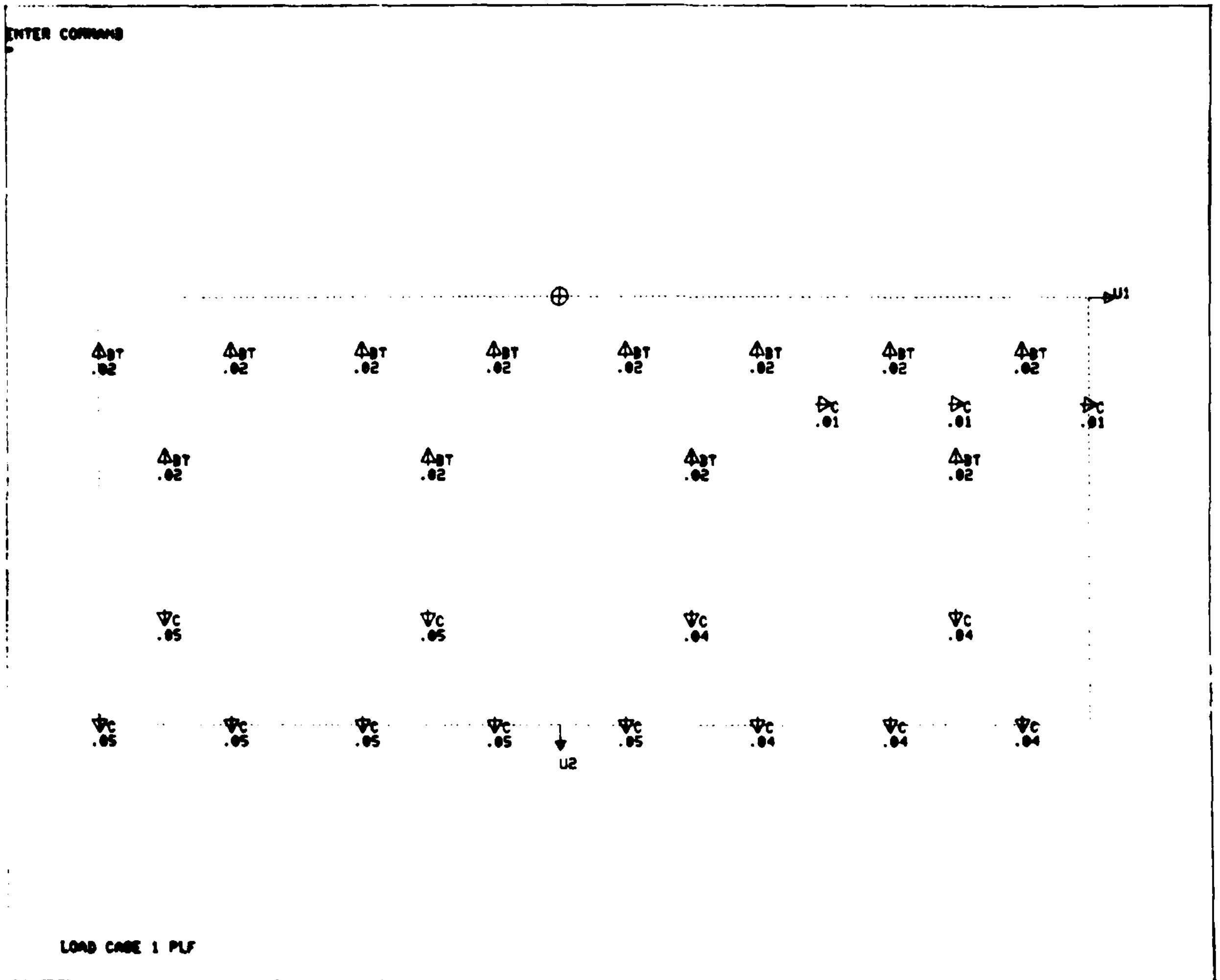

Figure B3. Display of the pile load factor and the PLF flag for each pile, for the current load case. Enter command PLF to obtain this diagram

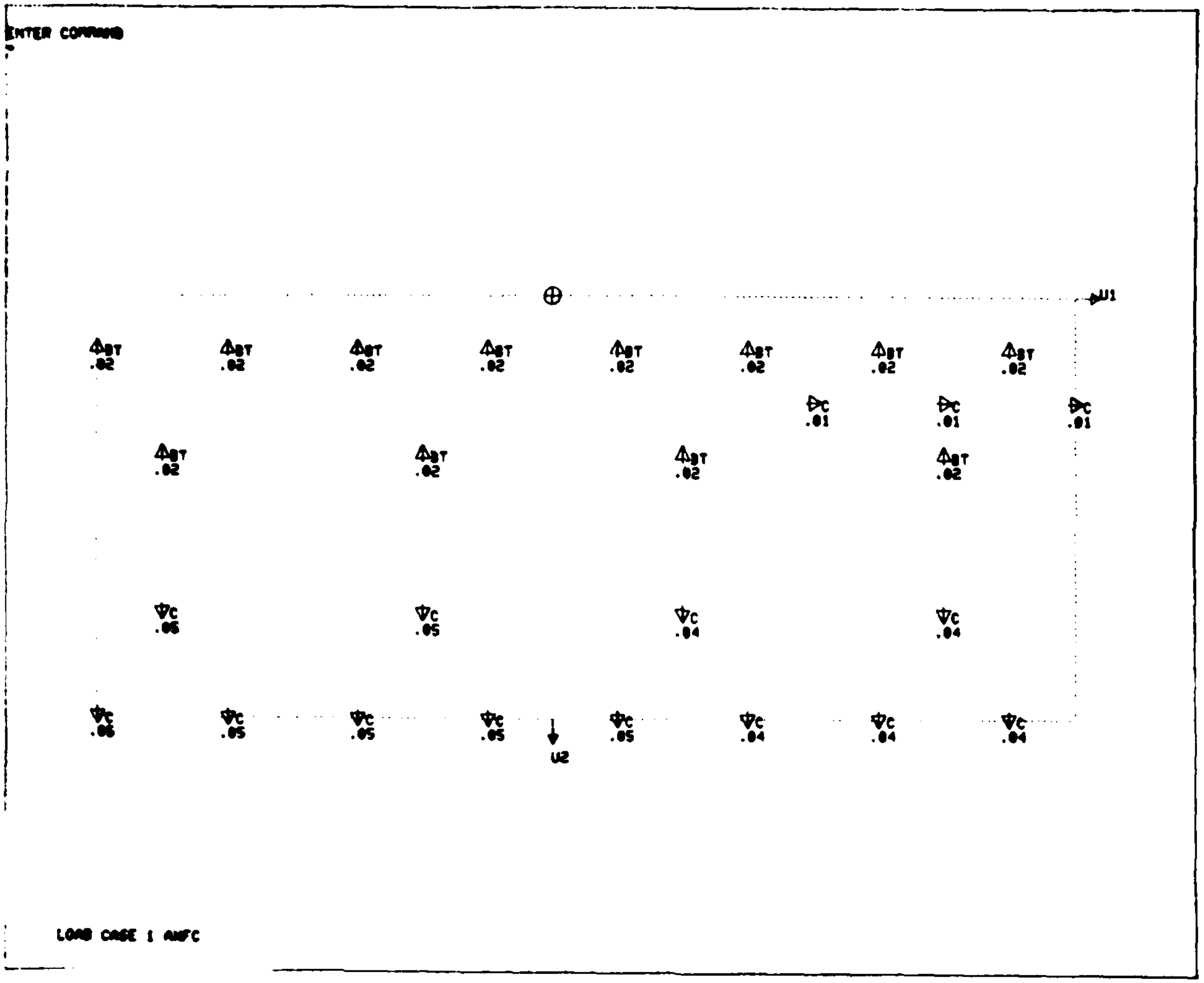

Figure B4. Display of the axial factor and the pile load factor flag for each pile, for the current load case. Enter the command AXFC to obtain this diagram

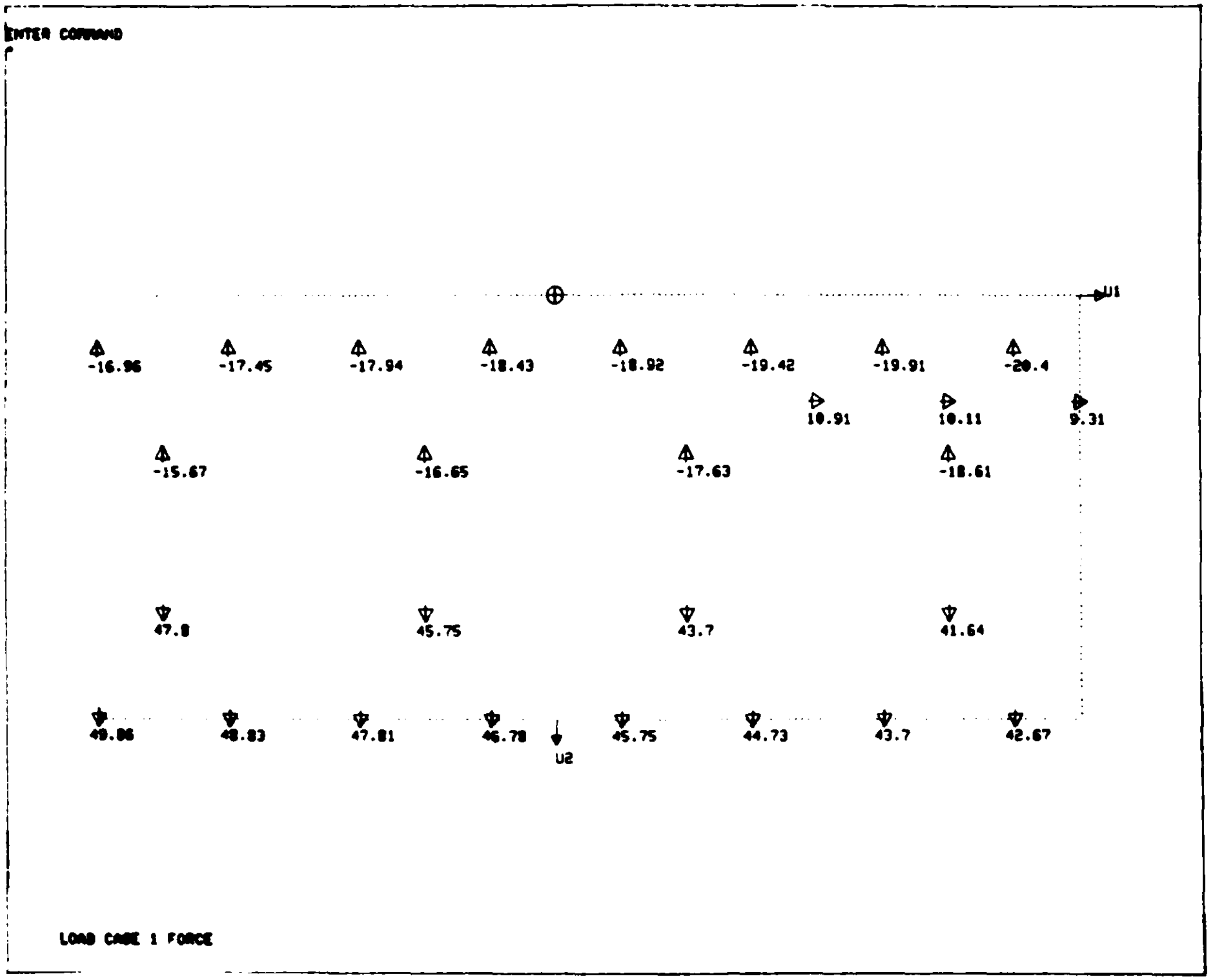

Figure B5. Display of the axial force Q3 for each pile for the current load case. Enter the command FORCE to obtain this diagram

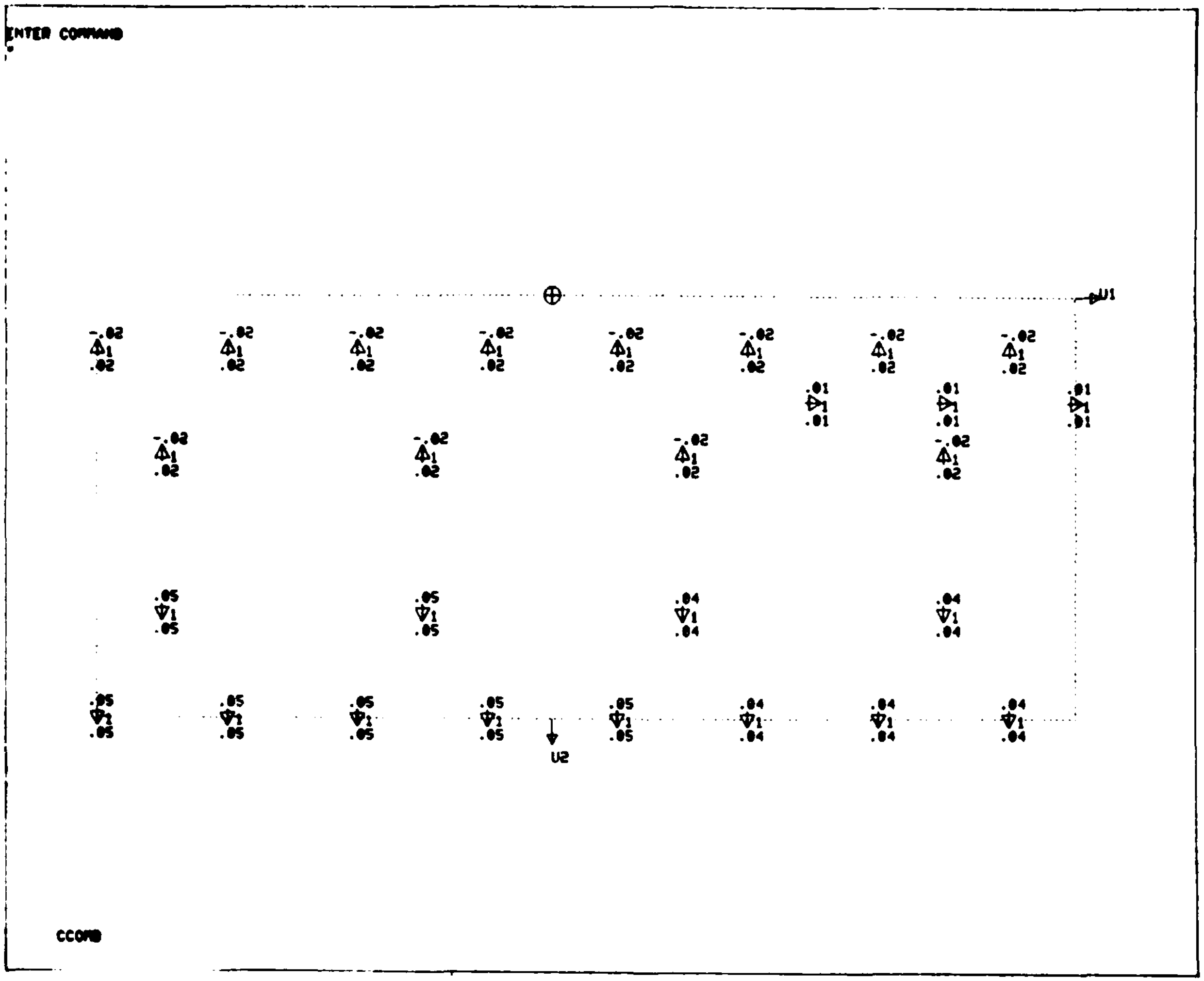

Figure B6. Display of the critical combined bending factor for all load cases and the critical load case numbers, for all piles. Enter the command CCOMB to obtain this diagram

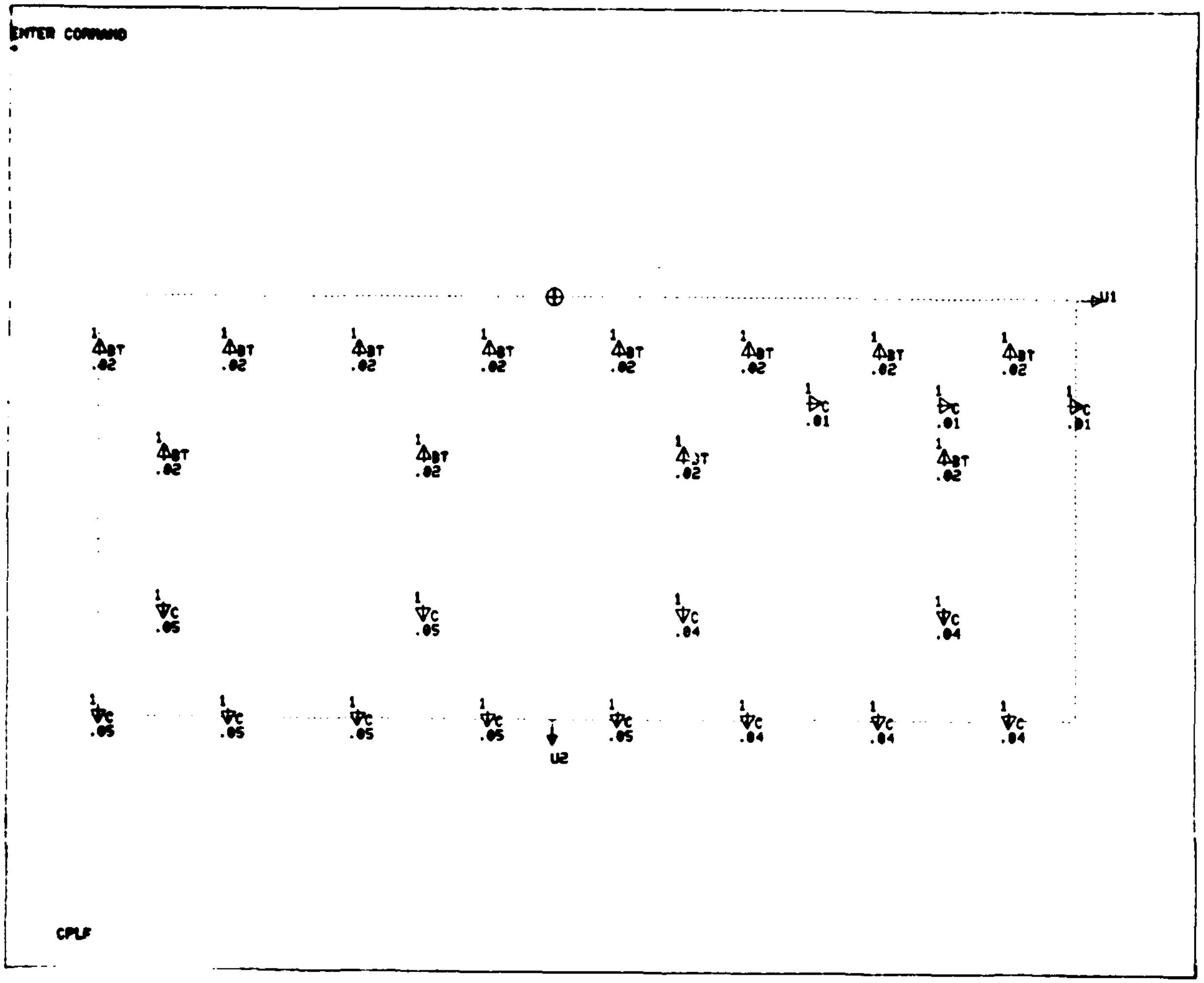

Figure B7. Display of the critical pile load factor for all load cases and the critical load case number, for each pile. Enter the command CPLF to obtain this diagram

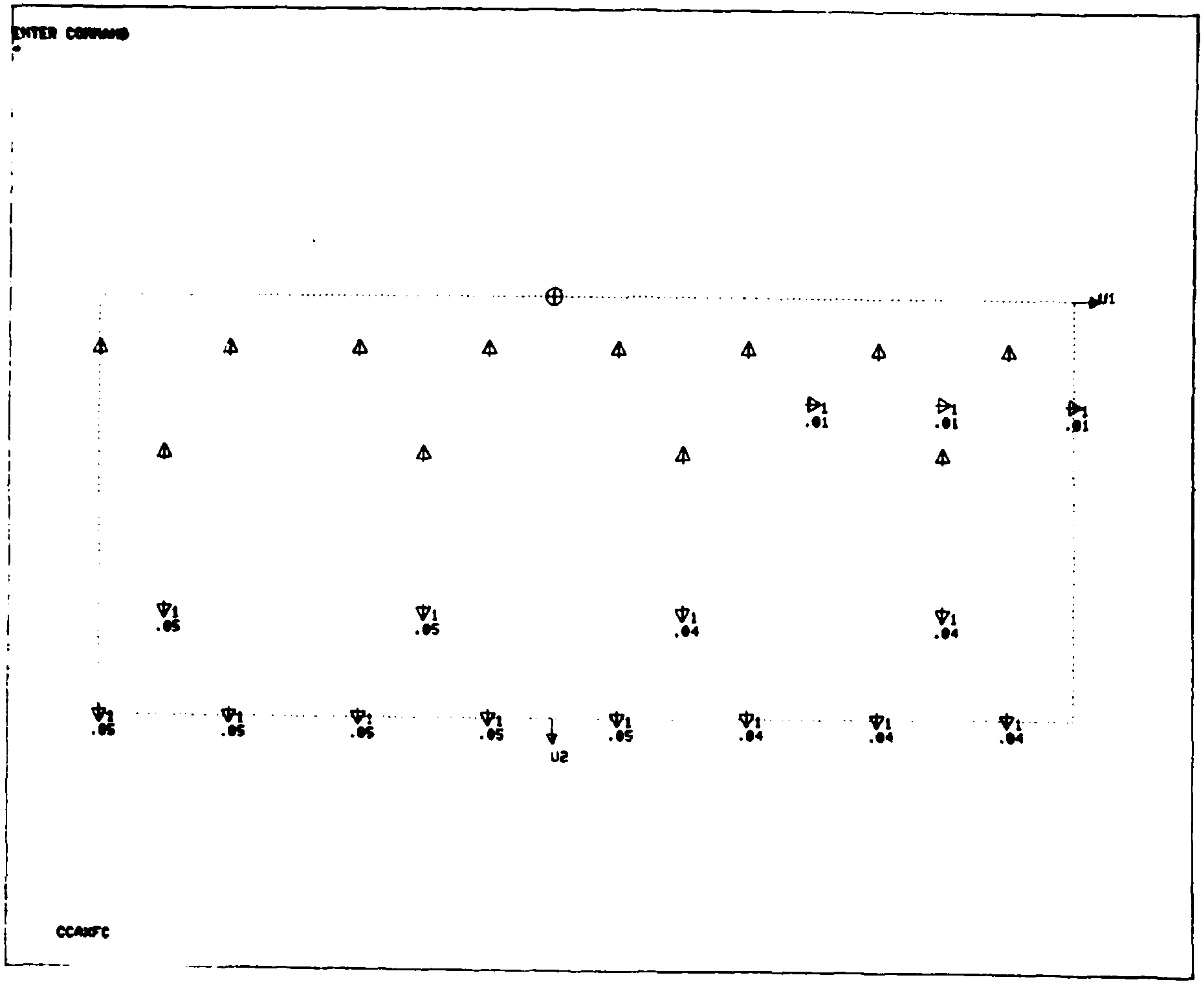

Figure B8. Display of the critical axial factor for all load cases and the critical load case number for each pile in compression. Enter the command CCAXFC to obtain this diagram

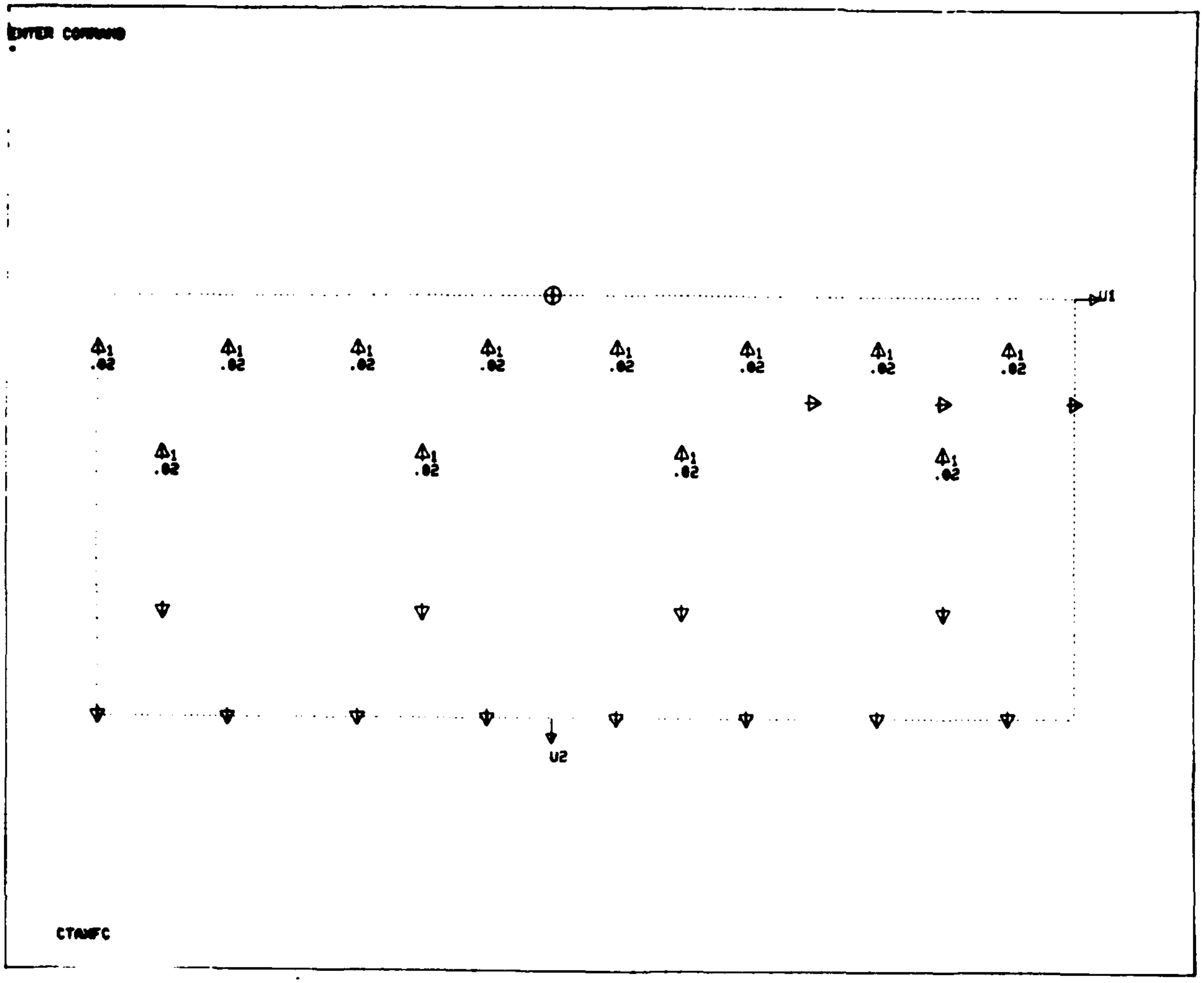

Figure B9. Display of the critical axial factor for all load cases and the critical load case number for each pile in tension. Enter the command CTAXFX to obtain this diagram

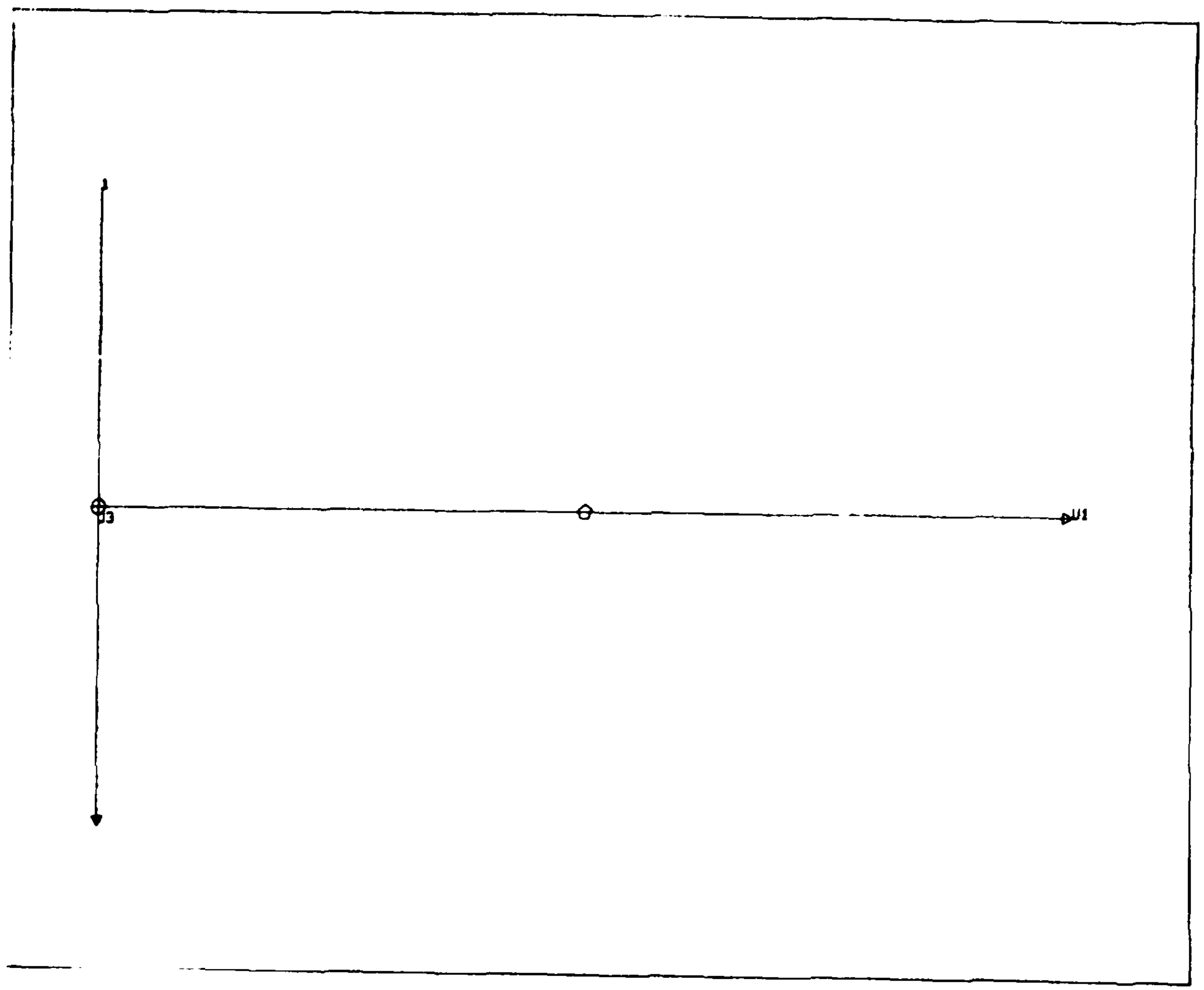

Figure B10. Display of the elastic center and the resultant force for load case one in the U1-U3 plane. Enter the command ELCEN to obtain this diagram and the ones shown in Figures B11 and B12

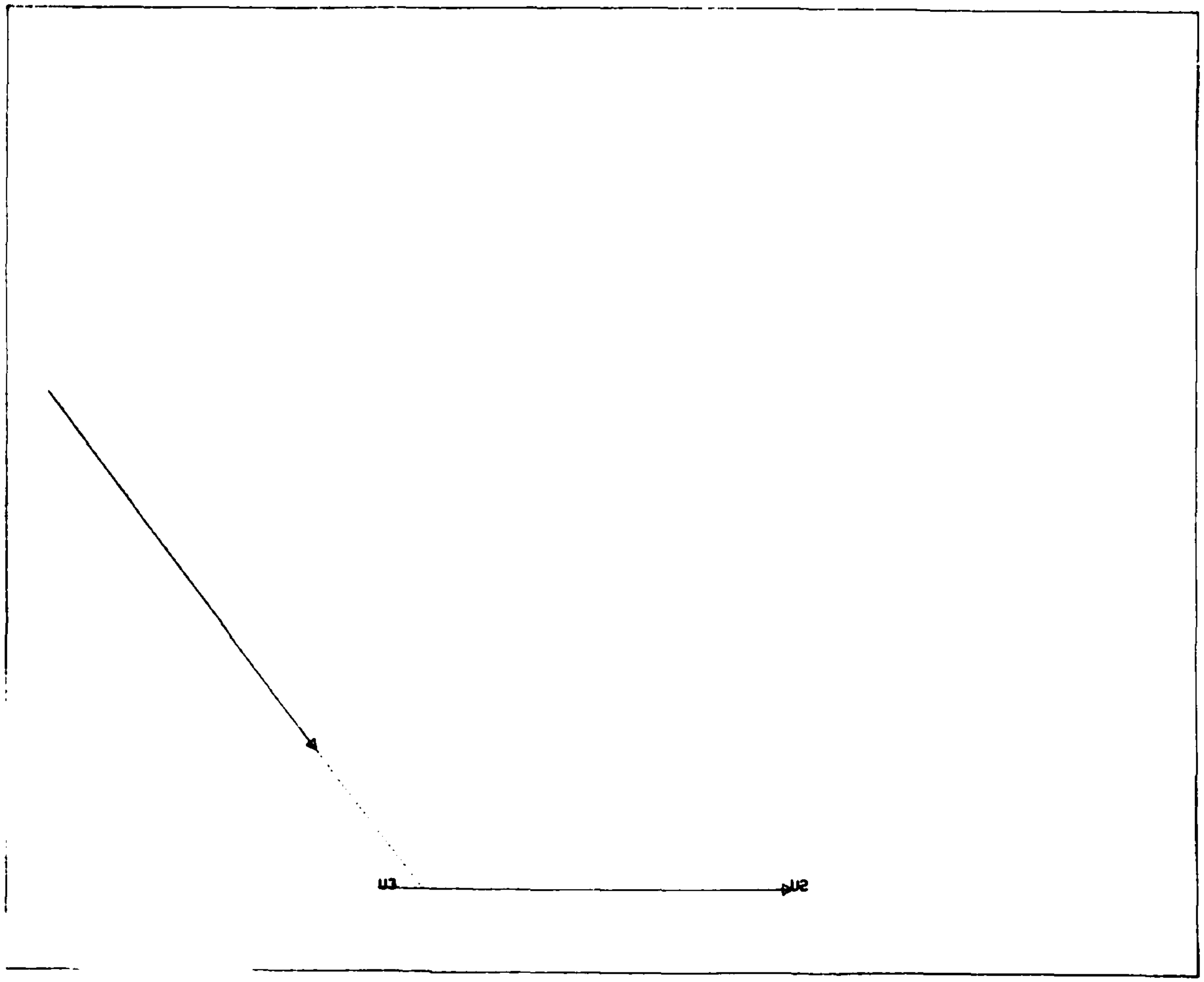

Figure B11. Display of the elastic center and resultant forces for load case one in the U2-U3 plane. Enter command ELCEN to obtain this diagram and the ones in Figures B10 and B12

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ENTER COMMAND
*

      RESULTANT FORCE DIAGRAMS SUMMARY

LC        Q1            Q3           QR           U1
 1           0.           344.90       344.90          0.

LC        Q2            Q3           QR           U2
 1          276.96        344.90       442.34        15.33
```

Figure B12. Display of the summary of the resultant force diagrams. Enter command ELCEN to obtain this diagram and the ones in Figures B10 and B11

In accordance with letter from DAEN-RDC, DAEN-ASI dated 22 July 1977, Subject: Facsimile Catalog Cards for Laboratory Technical Publications, a facsimile catalog card in Library of Congress MARC format is reproduced below.

Martin, Deborah K
  Documentation for LMVDPILE program / by Deborah K. Martin, H. Wayne Jones, N. Radhakrishnan. Vicksburg, Miss. : U. S. Waterways Experiment Station ; Springfield, Va. : available from National Technical Information Service, 1980.
  132, 38, 18 p. : ill. ; 27 cm. (Technical report - U. S. Army Engineer Waterways Experiment Station ; K-80-3)
  Prepared for U. S. Army Engineer Division, Lower Mississippi Valley, Vicksburg, Miss.
  References: p. 132.

1. Computer programs. 2. Computerized simulation. 3. Documentation. 4. Matrix analysis. 4. LMVDPILE (Computer program). 5. Pile caps. 6. Pile foundations. I. Jones, H. Wayne, joint author. II. Radhakrishnan, Narayanswamy, joint author. III. United States. Army. Corps of Engineers. Lower Mississippi Valley Division. IV. Series: United States. Waterways Experiment Station, Vicksburg, Miss. Technical report ; K-80-3.
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